28. On Persson's Theorem Concerning p-Nuclear Operators

By Yasuji TAKAHASHI*) and Yoshiaki OKAZAKI**)

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1. Let E, F be Banach spaces, p a real number such that $1\leqq p<\infty$ and $1/p+1/p'=1$. We denote by $N_p(E, F)$ the set of all linear operators T from E into F which can be factorized as follows:

$$(*)\qquad E\xrightarrow{V} l^\infty \xrightarrow{D} l^p \xrightarrow{W} F$$

where V, W are bounded linear operators and $D=(\alpha_n)$ is a diagonal operator with $\sum_n|\alpha_n|^p<\infty$. The elements in $N_p(E, F)$ will be called p-nuclear operators or operators of type N_p. We also denote by $N^p(E, F)$ the set of all linear operators T from E into F which can be factorized as follows:

$$(**)\qquad E\xrightarrow{V} l^{p'} \xrightarrow{D} l^1 \xrightarrow{W} F$$

where V, W and D are of the same kind as above. The elements in $N^p(E,F)$ will be called operators of type N^p. For $p=1$ the two classes $N_p(E, F)$ and $N^p(E,F)$ are equal and coincide with the space of all nuclear operators from E into F. For $1<p<\infty$ in general, neither $N_p(E, F)\subset N^p(E, F)$ nor the converse inclusion hold. In [3], Persson investigated some relation of these operators with p-integral and p-decomposable operators, and then proved that the inclusions $N^p(E, L^p)\subset N_p(E, L^p)$ and $N_p(L^{p'}, E)\subset N^p(L^{p'}, E)$ always hold for all Banach spaces E.

The purpose of this paper is to characterize Banach spaces E for which one of the following conditions holds:

(1) For each Banach space F, the inclusion $N^p(F, E)\subset N_p(F, E)$ holds.

(2) For each Banach space F, the inclusion $N_p(E, F)\subset N^p(E, F)$ holds.

We note that our results extend the works of Persson [3] and Kwapien [1]. As a consequence, we obtain that if E is of $S_{p'}$ type and F is of Q_p type in the sense of Kwapien [1], then the identity $N^p(E, F)=N_p(E, F)$ holds.

2. Main results. First we establish the relationship between p-nuclear operators and operators of type N^p. Throughout the paper, E denotes a Banach space with the dual E' and let p be $1\leqq p<\infty$. In the following, $\{e_n\}$ denotes the sequence of canonical basis of $l^{p'}$, where $1/p+1/p'=1$.

Theorem 1. *Let T be a bounded linear operator from E into a Banach space F. Then we have the following.*

(1) *If T is p-nuclear, then T' (dual of T) is of type N^p.*

(2) *If T is of type N^p, then T' is p-nuclear.*

*) Department of Mathematics, Yamaguchi University.
**) Department of Mathematics, Kyushu University.