26. On the Spectral Manifolds of the Simple Unilateral Shift and its Adjoint

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Summary. The concept of the "spectral manifold" is introduced by M. Radjabalipour [4] as a generalization or a modification of the spectral maximal space. In this paper, we show that the spectral manifolds of the simple unilateral shift S_0 on H^2 are only $\{0\}$ and H^2. And also we investigate some properties of the spectral manifolds of S_0^*.

1. Preliminaries. For a bounded linear operator T on the complex Banach space X, let

$$\sigma_p^0(T)=\{\lambda \in \boldsymbol{C} : (\omega I-T)f(\omega)\equiv 0 \text{ for some non-zero analytic function } f;\ D_r(\lambda)\to X\},$$

where $D_r(\lambda)=\{\omega\in \boldsymbol{C}: |\omega-\lambda|<r\}$ for some $r>0$. In case where $\sigma_p^0(T)=\phi$, T is said to have the single-valued extension property. For a closed set $\sigma\subset \boldsymbol{C}$, let

$$X_T(\sigma)=\{x\in X : (\omega I-T)f(\omega)\equiv x \text{ for some analytic function } f;\ \boldsymbol{C}\backslash\sigma\to X\},$$

and let $X_T(\tau)=\cup\{X_T(\sigma) : \sigma\subset\tau \text{ and } \sigma \text{ is closed}\}$ for an arbitrary set $\tau\subset\boldsymbol{C}$. The set $X_T(\tau)$ is called the spectral manifold of T.

The following proposition is immediate.

Proposition 1.

(i) *$X_T(\tau)$ is a hyper-invariant* (i.e., *invariant under every operator which commutes with T*) *linear manifold of T.*

(ii) *If $\tau_1\subset\tau_2$, then $X_T(\tau_1)\subset X_T(\tau_2)$.*

(iii) *$X_T(\tau)=X_T(\tau\cap\sigma(T))$, $X_T(\sigma(T))=X$ and $X_T(\phi)=\{0\}$.*

(iv) *$X_T(\sigma)\subset\bigcap_{\omega\notin\sigma}(T-\omega I)X$ for any closed set $\sigma\subset\boldsymbol{C}$.*

Proposition 2. *If $X_T(\tau)$ is closed, then we have $\sigma(T|X_T(\tau))\subset\tau\cup\sigma_p^0(T)^\sim$ where "$\sim$" denotes the closure.*

Proof. If $x\in X_T(\tau)$, then $x\in X_T(\sigma)$ for some closed set $\sigma\subset\tau$ and $(\omega I-T)f(\omega)\equiv x$ for some analytic function f; $\boldsymbol{C}\backslash\sigma\to X$. Since $f(\omega)\in X_T(\sigma)$ for any $\omega\in\boldsymbol{C}\backslash\sigma$ by [1], $x\equiv(\omega I-T)f(\omega)\in(\omega I-T)X_T(\sigma)\subset(\omega I-T)X_T(\tau)$ for any $\omega\in\boldsymbol{C}\backslash\sigma\supset\boldsymbol{C}\backslash\tau$. By the assumption and by Proposition 1, $X_T(\tau)$ is a closed invariant subspace of T and hence $X_T(\tau)=(\omega I-T|X_T(\tau))X_T(\tau)$ for any $\omega\in\boldsymbol{C}\backslash\tau$. Next, if $(\lambda I-T)x=0$ for any $\lambda\in\boldsymbol{C}\backslash[\tau\cup\sigma_p^0(T)^\sim]$ and for some $x\in X_T(\tau)$, then $x\in X_T(\sigma)$ for some closed set $\sigma\subset\tau$ and hence $(\omega I-T)f(\omega)\equiv x$ for some analytic function f; $\boldsymbol{C}\backslash\sigma\to X$. Since $(\omega I-T)[f(\omega)-(\omega-\lambda)^{-1}x]=(\omega I-T)f(\omega)-(\omega I-\lambda I+\lambda I-T)(\omega-\lambda)^{-1}x=x-x-(\omega-\lambda)^{-1}(\lambda I-T)x=0$ on $\boldsymbol{C}\backslash[\sigma\cup\{\lambda\}]$, $f(\omega)$