21. On Polarized Manifolds of Sectional Genus Two

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Let L be an ample line bundle on a compact complex manifold M of dimension n. Then the *sectional genus* of the polarized manifold (M, L) is given by the formula

$$2g(M, L)-2=(K+(n-1)L)L^{n-1},$$

where K is the canonical bundle of M. We have a satisfactory classification theory of polarized manifolds with $g(M, L)\leqq 1$ (see [1]). In this note we study the case $g(M, L)=2$. Details and proofs will be published elsewhere.

Definition. Let (M, L) be a polarized manifold and let p be a point on M. Let $\pi: M'\to M$ be the blowing-up at p and set $L'=\pi^*L-E$, where E is the exceptional divisor. If L' is ample, the polarized manifold (M', L') is called the *simple blowing-up* of (M, L) at p. Note that $g(M', L')=g(M, L)$ and $(L')^n=L^n-1$ in this case.

Theorem A. *Let (M, L) be a polarized manifold with $g(M, L)=2$, $n\geqq 3$ and $d=L^n>0$. Then one of the following conditions is satisfied:*

1) $K=(3-n)L$ *in* $\mathrm{Pic}(M)$ *and* $d=1$.

2) *M is a double covering of $\boldsymbol{P}^n$ with branch locus being a smooth hypersurface of degree* 6, *and L is the pull-back of $\mathcal{O}(1)$.* $d=2$.

2′) *(M, L) is a simple blowing-up of another polarized manifold (M_0, L_0) of the above type* 2). $d=1$ *and* $n=3$.

3) *There is a vector bundle $\mathcal{E}$ on a smooth surface S such that $M\simeq \boldsymbol{P}_S(\mathcal{E})$ and L is the tautological line bundle $\mathcal{O}(1)$.*

4) *There is a vector bundle $\mathcal{E}$ on a smooth curve C of genus two such that $M\simeq \boldsymbol{P}_C(\mathcal{E})$ and $L=\mathcal{O}(1)$.*

5) *There is a surjective morphism $f: M\to C$ onto a smooth curve C such that any fiber F of f is a hyperquadric in $\boldsymbol{P}^n$ and $L_F=\mathcal{O}_F(1)$.*

For a proof, we use the polarized version of Mori-type theory in [1]. The above conditions 2), 2′) and 4) are descriptive enough, so we will study the case 1), 3) and 5) in the sequel.

Theorem B. *Let (M, L) be a polarized manifold as in Theorem* A, 5). *Then there is a vector bundle $\mathcal{E}$ on C such that M is embedded in $P=\boldsymbol{P}_C(\mathcal{E})$ as a divisor, L is the restriction of the tautological line bundle H on P and $M\in|2H+\pi^*B|$ for some $B\in \mathrm{Pic}(C)$, where π is the projection $P\to C$. Moreover $h^1(C, \mathcal{O}_C)=0$ or* 1. *Set $b=\deg(B)$. Then:*

b0) *If $C\simeq \boldsymbol{P}^1$, then one of the following conditions is valid.*