20. On Automorphism Groups of Compact Riemann Surfaces of Genus 4

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Let X be a compact Riemann surface of genus $g \geqq 2$. A group AG of automorphisms of X (i.e., a subgroup of the group $\mathrm{Aut}(X)$ of all automorphisms of X) can be represented as a subgroup $R(X, AG)$ of $GL(g, \boldsymbol{C})$ as elements of AG operate in the g-dimensional module of abelian differentials on X. The purpose of this paper is to determine in case $g=4$ all subgroups of $GL(g, \boldsymbol{C})$ which are conjugate to some $R(X, AG)$ (for some X and some AG). (For the case $g=2, 3$ the same problem was already solved; [2] for the case $g=2$; the result for $g=3$ is not yet published.)

A more detailed account will be published elsewhere.

§1. Preliminaries. Let G be a finite subgroup of $GL(g, \boldsymbol{C})$, and H a non-trivial cyclic subgroup of G. Define two sets $CY(G)$ and $CY(G; H)$ by

$CY(G) := \{K; K \text{ is a non-trivial cyclic subgroup of } G\}$,

$CY(G; H) := \{K \in CY(G); K \text{ contains strictly a subgroup } H \text{ of } G\}$.

We say that G satisfies the condition (F) if for every element A of G, $r(A) := 2-(\mathrm{Tr}(A)+\mathrm{Tr}(A^{-1}))$ is a non-negative integer. Further we define as follows:

(1) $r(H) := 2-(\mathrm{Tr}(A)+\mathrm{Tr}(A^{-1}))$, where $H=\langle A\rangle$.

(2) $r_*(H) := r(H) - \sum_K r_*(K)$ (defined by descending condition)

where K ranges over the set $CY(G; H)$.

(3) $l(H) := r_*(H)/[N_G(H) : H]$, $l(I) := 0$, where I is the trivial group.

(4) $$g_0(G) := (1/\sharp G) \sum_{A \in G} \mathrm{Tr}(A).$$

Then we have the following relation [2]:

(RH) $$2g-2 = \sharp G(2g_0-2) + \sharp G \sum_i l(H_i)(1-(1/n_i)).$$

Here $\{H_i\}$ is a complete set of representatives of G-conjugacy classes of $CY(G)$ and $n_i := \sharp H_i$. We put further $\sharp G = n$.

We say that a finite subgroup G of $GL(g, \boldsymbol{C})$ satisfies (RH_+) if G satisfies (F) and if $l(H)$ is a non-negative integer for any H of $CY(G)$. Then put $\mathrm{RH}(G) := [g_0, n; n_1, \cdots, n_1, \cdots, n_s, \cdots, n_s]$, where n_i appears $l(H_i)$-times ($1 \leqq i \leqq s$).

We say that a finite subgroup G of $GL(g, \boldsymbol{C})$ satisfies the condition (E) if the following conditions are satisfied:

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