19. On the Construction of Pure Number Fields of Odd Degrees with Large 2-class Groups*)

By Shin NAKANO
Department of Mathematics, Gakushuin University

(Communicated by Shokichi IYANAGA, M. J. A., Feb. 12, 1986)

Introduction. In his previous paper [3], the author constructed infinitely many pure number fields of any given odd degree $n(>1)$ whose ideal class groups have 2-rank at least $2\Delta_n$, where Δ_n is the number of divisors of n which are smaller than n, that is $\Delta_n=\prod_{i=1}^{r}(e_i+1)-1$ if $n=\prod_{i=1}^{r}p_i^{e_i}$ is the decomposition of n into prime factors. The aim of the present paper is to give a stronger result. We shall namely show the following

Theorem. *For any odd natural number n greater than* 1, *there exist infinitely many pure number fields of degree n whose ideal class groups have* 2*-rank at least* $3\Delta_n$.

In order to prove this, we make use of the symmetric polynomial in X, Y, Z;

$$D(X,Y,Z)=\frac{X^2+Y^2+Z^2}{4}-\frac{XY+YZ+ZX}{2}$$
$$=\left(\frac{-X+Y+Z}{2}\right)^2-YZ=\left(\frac{X-Y+Z}{2}\right)^2-ZX$$
$$=\left(\frac{X+Y-Z}{2}\right)^2-XY.$$

Putting $(X,Y,Z)=(x^n,y^n,z^n)$ and A_i, C_i as in the table below, we obtain the polynomial $D(x^n,y^n,z^n)=C_1^2-A_1^n=C_2^2-A_2^n=C_3^2-A_3^n$.

i	A_i	$2C_i$
1	yz	$-x^n+y^n+z^n$
2	zx	$x^n-y^n+z^n$
3	xy	$x^n+y^n-z^n$

This polynomial, which will play an important part in our proof, is also applied to the research on "n-rank" of the ideal class groups of quadratic fields (Yamamoto [4], Craig [1], [2]). In that case, all the three above expressions of $D(x^n,y^n,z^n)$ cannot be used effectively (see [1] pp. 451). However, in the proof of our theorem, we take full advantage of them.

In case $n=3$ i.e. pure cubic case, corresponding to Craig's precise result [2] on 3-rank of the ideal class groups of quadratic fields, we can prove a 2-rank theorem giving a better estimation than above, which will appear elsewhere.

*) Partially supported by the Fûjukai Foundation.