18. On the Heat Operators of Cuspidally Stratified Riemannian Spaces

By Masayoshi NAGASE
Department of Mathematics, Tokyo Institute of Technology
School of Mathematics, The Institute for Advanced Study, Princeton, U.S.A.

(Communicated by Kôsaku YOSIDA, M. J. A., Feb. 12, 1986)

Introduction and statement of the Main Theorem. In this paper, we intend to examine a property of the Laplacian Δ of generalized Neumann or Dirichlet types ([2]) acting on the space of square-integrable forms on certain *incomplete* Riemannian manifolds called *cuspidally stratified Riemannian spaces* (briefly, *CSR-spaces*).

Main Theorem. *The heat operator $e^{-\Delta t}$ on a real n-dimensional CSR-space is of trace class and there exists a constant $K>0$ such that*

$$\mathrm{Tr}\, e^{-\Delta t} \leqq K t^{-n/2}, \qquad 0<t\leqq t_0.$$

The author's study of CSR-spaces was motivated by the desire to prove a similar result for the smooth part $\mathcal{X}$ of a projective variety X (with the induced Fubini-Study metric, which is therefore incomplete). However such spaces do not fall into the category of CSR-spaces studied in this paper. In fact, even on a normal singular projective surface, the metric near the singular point is more complicated ([3]). Nevertheless the author believes that it will not be too difficult to extend the theory of CSR-spaces to projective varieties and that this will provide a suitable framework for studying the global analysis of singular projective varieties.

§ 1. Definition of CSR-spaces. Let X be a real n-dimensional compact stratified space (possibly with boundary) with Thom structure $\{\mathcal{T}, \mathcal{S}\}$ ([4]). Here $\mathcal{S}$ is the stratification of X (that is, a decomposition of X into smooth manifolds without boundaries) and $\mathcal{T}$ is a collection of open tubular neighborhoods of the strata (i.e., the elements of $\mathcal{S}$), where each open tubular neighborhood T_V ($V\in\mathcal{S}$) is endowed with the following three objects: the structure of a fibre bundle, $\pi_V : T_V \to V$, a so-called distance function from V, $\lambda_V : T_V \to [0, \infty)$, and a homeomorphism h_V from the mapping cylinder $M(\pi_V \,|\, \lambda_V^{-1}(1))$ to T_V. Note that $(T_V, \pi_V, \lambda_V, h_V)$, $V\in\mathcal{S}$, are compatible with each other in a natural sense.

Now let Σ be the (disjoint) union of the strata with positive codimensions and set $\mathcal{X}=X-\Sigma$. This manifold together with the metric g described below is called a *CSR-space*.

For each stratum $V\in\mathcal{S}$ with $\dim V<n$, let k_V be a real number with $k_V=0$ if $\dim V=n-1$ and $k_V\geqq 1$ if $\dim V<n-1$; set $\boldsymbol{k}=\{k_V : V\in\mathcal{S}, \dim V<n\}$. Then the metric g depends on $\boldsymbol{k}$ and is characterized near the strata with positive codimensions as follows: