12. *Automorphism Groups of Real Algebraic Curves of Genus 3*

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In [3] we have obtained the full list of automorphism groups of real (irreducible) algebraic hyperelliptic curves of genus 3. Let C be such a curve defined on $\boldsymbol{R}$ and $C(\boldsymbol{R})$ its real part. The automorphism group of $C(\boldsymbol{R})$, i.e., the group of its birational transformations, is one of the following: C_2, $C_2\times C_2$, $C_2\times C_2\times C_2$, D_6 and $C_2\times D_4$, with the notation of [5]. Even more, if we denote by k the number of connected components of a non-singular model of $C(\boldsymbol{R})$, the following table recollects the automorphism groups according to the topological features of the curves:

Table I

$C\backslash C(\boldsymbol{R})$	k	C_2	$C_2\times C_2$	$C_2\times C_2\times C_2$	D_6	$C_2\times D_4$
Non-connected	4	*	*	*		*
Non-connected	2	*	*	*	*	*
Connected	1	*	*			
Connected	2	*	*	*		
Connected	3	*	*		*	

The sign * means the actual occurrence while no sign does non-occurrence.

In this note we extend this result to arbitrary real algebraic curves of genus 3. The technique is based upon the well-known functorial equivalence between real algebraic curves and bordered compact Klein surfaces (see [1]): Given a curve C of genus g, Alling and Greenleaf endow it with a structure of Klein surface, that is, a compact surface $X(C)=\{V\mid V \text{ is a valuation ring of } \boldsymbol{R}(C), \boldsymbol{R}\subset V\}$. Its boundary, consisting of those residually real valuation rings, is the non-singular model of $C(\boldsymbol{R})$.

Now, a Klein surface X may be expressed as D/Γ, where D is the hyperbolic plane and Γ is a non-Euclidean crystallographic (NEC) group, i.e., a discrete subgroup of isometries of the hyperbolic plane with compact quotient. (See [6].)

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