109. *Isotropic Submanifolds in a Euclidean Space*

By Takehiro ITOH
Institute of Mathematics, Tsukuba University

(Communicated by Kôsaku YOSIDA, M. J. A., Dec. 12, 1986)

The Gauss map of a submanifold M in a Euclidean n-space E^n is the map which is obtained by the parallel displacement of the tangent plane of M in E^n. It is well known that the image of an m-dimensional submanifold in E^n by the Gauss map lies in the Grassmann manifold $G(m, n-m)$. The Gauss map is useful for the study of submanifolds in E^n.

In the present paper we will discuss isotropic submanifolds in E^n with conformal Gauss map and prove the following

Theorem. *Let M be an m-dimensional Riemannian manifold isotropically immersed in E^n. If the Gauss map Γ is conformal and the image $\Gamma(M)$ is totally umbilical in $G(m, n-m)$, then M is a minimal and isotropic submanifold in a hypersphere S^{n-1} of E^n with the parallel second fundamental form.*

We well know that minimal isotropic submanifolds in a sphere with the parallel second fundamental form are classified in [5].

§ 1. **Preliminaries.** In the present paper we use the notations introduced in [3] and [4]. Let M be an m-dimensional Riemannian manifold immersed in E^n through the isometric immersion ι. In each neighborhood $V \subset M$, M is given by differentiable functions

$$x^A = x^A(y^1, y^2, \cdots, y^m), \tag{1.1}$$

where x^A $(A=1, 2, \cdots, n)$ are rectangular coordinates of E^n and y^i $(i=1, 2, \cdots, m)$ local coordinates of M in V. We define B_i^A by $B_i^A = \partial x^A/\partial y^i$. The tangent plane $\iota(M_p)$, $p \in M$, of ιM may be considered as a point $\Gamma(p)$ of $G(m, n-m)$ by the parallel displacement in E^n, and so we get naturally a mapping $\Gamma: M \to G(m, n-m)$ which is called *the Gauss map* associated with the immersion ι and $\Gamma(M)$ *the Gauss image* of M. In the present paper, we always assume that *the Gauss map is regular*.

Now, we assume that $V \subset M$ is a neighborhood of a fixed point $p \in M$ whose local coordinates satisfy $y^i = 0$, $i=1, \cdots, m$. Let (e_i, e_α) be a fixed orthonormal frame of E^n such that e_i are vectors of $\iota(M_p)$ and e_α are normal to $\iota(M_p)$. For each point $q \in V$, let (f_i, f_α) be an orthonormal frame of E^n where f_i are vectors of $\iota(M_q)$ and f_α are normal to $\iota(M_q)$ such that, in V, (f_i, f_α) is a differentiable frame satisfying $\langle f_i, e_j\rangle = \langle f_j, e_i\rangle$, $\langle f_\alpha, e_\beta\rangle = \langle f_\beta, e_\alpha\rangle$ and $f_i(0) = e_i$, $f_\alpha(0) = e_\beta$. Denoting f_i^A the components of the vector f_i, we may put $f_i^A = \sum_k \gamma_i^k B_k^A$. The matrix (γ_j^i) satisfies $\sum \gamma_i^l \gamma_j^k g_{lk} = \delta_{ij}$, $g_{ij} = \sum B_i^A B_j^A$, where g_{ij} are the components of the first fundamental form g of M. Then we have $\sum \gamma_l^i \gamma_l^j = g^{ij}$ where $\sum g^{ik} g_{kj} = \delta_j^i$. The components of the second