108. *Note on Chemical Interfacial Reaction Models*

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§ 1. **Introduction.** We consider the following parabolic system with nonlinear boundary conditions. Find $u_i=u_i(x,t)$ $(i=1,2,3)$ satisfying

$$(\mathrm{P})\quad\begin{cases}(1)\quad a_i(x)\dfrac{\partial u_i}{\partial t}=\dfrac{\partial^2 u_i}{\partial x^2}, & x\in I\equiv(0,1), \quad t\in(0,\infty),\\[2mm] (2)\quad \dfrac{\partial u_i}{\partial x}(0,t)=R_i(u_1(0,t),\,u_2(0,t),\,u_3(0,t)), & t\in(0,\infty),\\[2mm] (3)\quad \dfrac{\partial u_i}{\partial x}(1,t)=0, & t\in(0,\infty),\\[2mm] (4)\quad u_i(x,0)=\varphi_i(x), & x\in I,\end{cases}$$

where a_i and R_i $(i=1,2,3)$ are given functions and φ_i $(i=1,2,3)$ are given initial data. Concerning a_i and R_i $(i=1,2,3)$, we introduce the following assumptions.

(A.1) $a_i\geqq 0$ on $\bar{I}\equiv[0,1]$, $a_i\in C^\infty(\bar{I})$, $a_i>0$ on $[0,1)$.

(R.1) $R_i(u_1,u_2,u_3)\in C^\infty(U)$, where U is an open set in $\boldsymbol{R}^3$ satisfying $U\supset[-\delta,\delta]^3\cup[0,\infty)^3$ with some positive constant δ.

(R.2) For any compact subset K of U, there exists a positive constant c_K such that

$$\sum_{i=1}^3 R_i(u_1,u_2,u_3)u_i^-\leqq c_K\sum_{i=1}^3|u_i^-|^2 \qquad \text{for any } (u_1,u_2,u_3)\in K,$$

where $u_i^-=-\min\{u_i,0\}$.

(R.3) There exist positive constants α_i $(i=1,2,3)$ such that

$$\sum_{i=1}^3\alpha_i R_i(u_1,u_2,u_3)=0 \qquad \text{for all } (u_1,u_2,u_3)\in U.$$

(R.4) There exists a positive constant c such that

$$-\sum_{i=1}^3 R_i(u_1,u_2,u_3)u_i^{2p-1}\leqq c\sum_{i=1}^3|u_i|^{2p}$$

$$\text{for all } (u_1,u_2,u_3)\in[0,\infty)^3 \text{ and } p\in[1,\infty).$$

Our motivation for (P) comes from the chemical interfacial model proposed by Kawano, Kusano, Kondo and Nakashio [3]. They analyzed the kinetics of interfacial reaction by comparing the chemical experiments with numerical simulation of the models. After suitable transformation, their typical model is reduced to (P) with the case,

$$(5)\quad \begin{aligned} &a_i(x)=k_i(1-x^2) \qquad (i=1,2,3),\\ &R_1=R_2=-R_3=\frac{u_1u_2-k_7u_3}{k_4+k_5u_2+k_6u_1u_2},\end{aligned}$$

where k_i $(i=1,\cdots,7)$ are positive constants determined by chemical substances. It is easily seen that the conditions (A.1), (R.1), (R.2), (R.3) and (R.4) are satisfied by taking

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