105. The 102-Velocity Model and the Related Discrete Models of the Boltzmann Equation

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In the preceding paper [1], we established the existence of infinitely many regular discrete models invariant under the transformation group $G=A_5\times I$, a finite subgroup of $O(3)$. All the models constructed in [1] have 8 linearly independent summational invariants. In this note, we show the following: There exist also infinitely many regular discrete models with the symmetry group $G=A_5\times I$ such that the associated spaces of summational invariants have 9 dimensions. All these models are normal in the sense that any summational invariant can be expressed in terms of the summational invariants issuing from the conservation laws of mass, momentum, and energy. Note that, so far as the models are normal and described by means of a certain quadratic field, the maximal number of the dimension of the space of summational invariants is 9. We study first the 102-velocity model defined by combining the 12-velocity model with the 90-velocity model. Then we construct a sequence of regular discrete models by induction.

1. We recall that the 90-velocity model is defined by using a dodecahedron with its center at the origin. Let $\tau=(1+\sqrt{5})/2$. Then, $\{1,\tau\}$ forms the standard basis of the quadratic field $Q(\sqrt{5})$. By means of this basis, the coordinates of $v_1,\cdots,v_{90}$ are expressed by sextuples of rational integers as follows.

$v_1=(0,2,0,0,0,0)$, $v_2=(0,0,0,2,0,0)$, $v_3=(0,0,0,0,0,2)$,
$v_7=(1,0,1,1,0,-1)$, $v_8=(-1,-1,0,1,1,0)$, $v_9=(0,1,1,0,1,1)$,
$v_{13}=(-1,0,1,1,0,-1)$, $v_{14}=(-1,-1,0,-1,-1,0)$, $v_{15}=(0,-1,1,0,1,1)$,
$v_{19}=(-1,0,-1,-1,0,-1)$, $v_{20}=(1,1,0,-1,1,0)$, $v_{21}=(0,-1,-1,0,1,1)$,
$v_{25}=(1,0,-1,-1,0,-1)$, $v_{26}=(1,1,0,1,-1,0)$, $v_{27}=(0,1,-1,0,1,1)$,
$v_i=-v_{i-3}$ for $i=j+6k-3$, $1\le j\le 3$, $1\le k\le 5$, $v_{31}=(0,2,0,0,0,2)$,
$v_{32}=(0,0,0,2,0,2)$, $v_{33}=(0,-2,0,0,0,2)$, $v_{34}=(0,0,0,-2,0,2)$,
$v_{39}=(0,2,0,2,0,0)$, $v_{40}=(0,-2,0,2,0,0)$, $v_{43}=(1,1,2,1,1,0)$,
$v_{44}=(-1,0,1,1,2,1)$, $v_{45}=(-1,1,0,-1,1,2)$, $v_{46}=(1,2,1,-1,0,1)$,
$v_{51}=(0,-1,1,2,1,-1)$, $v_{52}=(-2,-1,-1,0,1,1)$, $v_{55}=(-1,-1,2,1,1,0)$,
$v_{56}=(-1,-2,1,-1,0,1)$, $v_{57}=(1,-1,0,-1,1,2)$, $v_{58}=(1,0,1,1,2,1)$,
$v_{63}=(-2,-1,1,0,-1,-1)$, $v_{64}=(0,-1,-1,-2,-1,1)$,

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