102. *On Automorphisms of Algebraic K3 Surfaces which Act Trivially on Picard Groups*

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1. Introduction. In this note we study automorphisms of algebraic $K3$ surfaces over $\boldsymbol{C}$ which act trivially on Picard groups. Recall that a $K3$ *surface* X is a nonsingular compact complex surface with trivial canonical bundle and $\dim H^1(X, \mathcal{O}_X)=0$. The second cohomology group $H^2(X, \boldsymbol{Z})$ admits a canonical structure of a lattice of rank 22 induced from the cup product. We denote by S_X the Picard group of X. Then S_X has a structure of a sublattice of $H^2(X, \boldsymbol{Z})$. Let T_X be the orthogonal complement of S_X in $H^2(X, \boldsymbol{Z})$ which is called a *transcendental lattice* of X. Put $H_X=\mathrm{Ker}(\mathrm{Aut}(X)\to\mathrm{Aut}(S_X))$. Then H_X is a cyclic group $\boldsymbol{Z}/m$ of order m, and $\phi(m)$ is a divisor of the rank of T_X where ϕ is the Euler function ([3], Corollary 3.3).

Theorem. *Let X be an algebraic* $K3$ *surface and m_X the order of H_X. Assume that the lattice T_X is unimodular* (*i.e.* $\det(T_X)=\pm1$). *Then*

(1) *m_X is a divisor of* 66, 44 *or* 12.

(2) *Suppose that $\phi(m)=\mathrm{rank}(T_X)$. Then m_X is equal to either* 66 *or* 42. *Moreover for $m=66$ or* 42, *there exists a unique* (*up to isomorphisms*) *algebraic* $K3$ *surface with $m_X=m$.*

In case T_X is non unimodular, Vorontsov [8] proved a similar result as the above theorem. However his statement for unimodular case is not complete and contains a mistake, i.e. he claims that there exists an algebraic $K3$ surface with $m_X=12$ and $\mathrm{rank}(T_X)=\phi(12)$ (his proof has not yet published). His method is based on the theory of a cyclotomic field $\boldsymbol{Q}(m)$. Here we use only the theory of elliptic surfaces due to Kodaira [1].

2. Example. In this section we construct two examples of algebraic $K3$ surfaces with $m_X=66$, 42.

(2.1) **Example 1.** Let (x, y, z) be a system of a homogeneous coordinate of $\boldsymbol{P}^2$. We take two copies $W_0=\boldsymbol{P}^2\times C_0$ and $W_1=\boldsymbol{P}^2\times C_1$ of the cartesian product $\boldsymbol{P}^2\times \boldsymbol{C}$ and form their union $W=W_0\cup W_1$ by identifying $(x, y, z, u)\in W_0$ with $(x_1, y_1, z_1, u_1)\in W_1$ if and only if $u\cdot u_1=1$, $x=x_1$, $y=u_1^6\cdot y$ and $z=u_1^2\cdot z_1$. We define a subvariety X of W by the following equations:

$$(2.2)\qquad \begin{aligned} &z^3-y\left\{y^2\prod_{i=1}^{12}(u-\xi_i)-x^2\right\}=0,\\ &z_1^3-y_1\left\{y_1^2\prod_{i=1}^{12}(1-u_1\cdot\xi_i)-x_1^2\right\}=0\end{aligned}$$

where ξ_i $(i=1, 2, \cdots, 12)$ are distinct comlex numbers. Let π be a projection from X to the u-sphere $\boldsymbol{P}^1$. It is easy to see that X is non singular