99. *A Note on a Global Version of the Coleman Embedding*

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§ 1. **Introduction.** Let l be an odd prime number and $(\zeta_\nu)_{\nu\geq 1}$ be a fixed system of primitive l^ν-th root of unity with $\zeta_{\nu+1}^l=\zeta_\nu$. Let Ω_l^- be the "minus part" of the maximum pro-l abelian extension Ω_l over the cyclotomic field $\boldsymbol{Q}(\mu_{l^\infty})$ unramified outside l, and set $\mathfrak{G}=\mathrm{Gal}(\Omega_l^-/\boldsymbol{Q}(\mu_{l^\infty}))$. Let $\mathfrak{U}$ be the inertia group of an extension of l in $\Omega_l^-/\boldsymbol{Q}(\mu_{l^\infty})$, and let $\mathfrak{U}'$ be the projective limit of the principal unit group of $\boldsymbol{Q}_l(\zeta_\nu)$ w.r.t. the relative norm.

R. Coleman [1] constructed an embedding (w.r.t. the system $(\zeta_\nu)_\nu$) $[\mathrm{Col}]':\mathfrak{U}'\to\boldsymbol{Z}_l[[T]]^\times$, which is a basic tool in the theory of cyclotomic fields. By class field theory, $[\mathrm{Col}]'$ induces, naturally, an embedding $[\mathrm{Col}]:\mathfrak{U}\to\boldsymbol{Z}_l[[T]]^\times$. Under the conjecture $(\mathfrak{C})$ that $L_l(m,\omega^{1-m})\neq 0$ for any odd integer $m\geq 3$, we can extend $[\mathrm{Col}]$ to a homomorphism $\mathfrak{G}\to\boldsymbol{Q}_l[[T]]^\times$ as follows (where ω denotes the Teichmüller character and $L_l(s,\omega^{1-m})$ denotes the l-adic L-function): Note that for $\rho\in\mathfrak{U}$,

$$[\mathrm{Col}](\rho)=\exp\left(\sum_{\substack{m\geq 3\\ \mathrm{odd}}}\frac{\varphi_m(\rho)}{m\,!}X^m\right)$$

where φ_m is the Coates-Wiles homomorphism and $X=\log(1+T)$. Let χ_m be the Kummer character w.r.t. the system of the l-units

$$\varepsilon_\nu(m)=\prod_{\substack{1\leq a\leq l^\nu\\ (a,l)=1}}(\zeta_\nu^a-1)^{a^{m-1}},$$

i.e. χ_m is a homomorphism $\mathfrak{G}\to\boldsymbol{Z}_l$ such that

$$(\varepsilon_\nu(m)^{1/l^\nu})^{\rho-1}=\zeta_\nu^{\chi_m(\rho)}$$

for any $\nu\geq 1$ and $\rho\in\mathfrak{G}$. This Kummer character is considered in Soulé [8], Deligne [3] and Ihara [5]. See, also, Ichimura-Sakaguchi [4]. By Coleman, $\chi_m|\mathfrak{U}=(1-l^{m-1})L_l(m,\omega^{1-m})\varphi_m$. Therefore, under the conjecture $(\mathfrak{C})$, the homomorphism

$$\psi:\mathfrak{G}\ni\rho\mapsto f_\rho(T)=\exp\left(\sum_{\substack{m\geq 3\\ \mathrm{odd}}}\frac{(1-l^{m-1})^{-1}L_l(m,\omega^{1-m})^{-1}\chi_m(\rho)}{m\,!}X^m\right)\in\boldsymbol{Q}_l[[T^\times]]$$

is a global version of $[\mathrm{Col}]$, i.e. $\psi|\mathfrak{U}=[\mathrm{Col}]$.

The purpose of this note is to study some properties of ψ. Clearly, $\psi^{-1}(\boldsymbol{Z}_l[[T]]^\times)\supset\mathfrak{U}\,\mathrm{Ker}\,\psi$. But since there appear $L_l(m,\omega^{1-m})^{-1}$ in the coefficient of T^m of $f_\rho(T)$, there may be some $\rho\in\mathfrak{G}$ such that $f_\rho(T)\notin\boldsymbol{Z}_l[[T]]^\times$. The main aim of this note is to show the following

Theorem (Under the conjecture $(\mathfrak{C})$). $\psi^{-1}(\boldsymbol{Z}_l[[T]]^\times)=\mathfrak{U}\,\mathrm{Ker}\,\psi$.

Further, we prove a proposition on the kernel of ψ.

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