98. On Closed Maximal Ideals of M*),†)

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1. Introduction. Let U be the unit disc $\{|z|<1\}$ in $\boldsymbol{C}$. A function f holomorphic in U is said to belong to the class M if

$$\int_0^{2\pi} \log^+ Mf(\theta)\frac{d\theta}{2\pi}<\infty,$$

where $Mf(\theta)=\sup_{0\le r<1}|f(re^{i\theta})|$ and $\log^+ x=\max(\log x, 0)$, $x>0$. The class M was introduced and studied in [3]. It is shown that

$$\bigcup_{p>0} H^p \subsetneqq M \subsetneqq N^+,$$

where H^p is the usual Hardy class of order $p>0$ and N^+ the Smirnov class. See [1] or [2] for the general theory of H^p and N^+.

The space M with the metric given by

$$d(f, g)=\int_0^{2\pi} \log(1+M(f-g)(\theta))\frac{d\theta}{2\pi}$$

is an F-algebra, i.e., a topological vector space with a complete translation invariant metric in which multiplication is continuous. The class M has many similarities with the Smirnov class N^+ as an F-algebra. See [3] and [4]. For example, the following are noted in [3].

(1) For $\lambda\in U$, if we define

$$\gamma_\lambda(f)=f(\lambda), \qquad f\in M,$$

then γ_λ is a continuous multiplicative linear functional on M. Conversely, if γ is a nontrivial multiplicative linear functional on M then $\gamma=\gamma_\lambda$ for some $\lambda\in U$.

(2) If $\lambda\in U$ and $m_\lambda=\{f\in M: f(\lambda)=0\}$ then $m_\lambda=(z-\lambda)M$ and m_λ is a closed maximal ideal of M.

(3) There exists a maximal ideal m of M which is not the kernel of a multiplicative linear functional on M.

In this note, we show that every closed maximal ideal is the kernel of a multiplicative linear functional on M (see Corollary 5). The corresponding theorem for N^+ was proved [4].

2. Main theorem.

Lemma 1. *Let m be a nonzero ideal of M. Then m contains a bounded holomorphic function which is not identically zero.*

Proof. Let $f\in m$ and $f\not\equiv 0$. Since $M\subset N^+$, f can be factored canoni-

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