95. *Uniqueness of the ω-limit Point of Solutions of a Semilinear Heat Equation on the Circle*

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§ 1. **Introduction.** In this paper we discuss the asymptotic behavior of a semilinear heat equation on the circle. Namely we shall show that the ω-limit set of any solution contains at most one element. This implies that any bounded global solution converges to an equilibrium solution as $t\to\infty$.

To be more precise, consider the initial value problem

$$(1.1)\qquad \begin{cases} u_t=u_{xx}+f(u), & x\in \boldsymbol{R},\ t>0,\\ u(x,0)=u_0(x), & x\in \boldsymbol{R},\\ u_0(x+1)=u_0(x), & x\in \boldsymbol{R}, \end{cases}$$

where f is a C^1-function on $\boldsymbol{R}$ and u_0 is a continuous function of period 1. It is well known that (1, 1) has a unique classical solution $u(x,t)$ such that $u(\cdot,t)\in L^\infty(\boldsymbol{R})$ for every $t\in[0,s(u))$, where $[0,s(u))$ denotes the maximal interval of existence for the solution u. Also one can easily show that $u(x+1,t)=u(x,t)$ for $x\in\boldsymbol{R}$, $t>0$. Therefore (1.1) can be regarded as an equation on a circle.

For a solution u of (1.1), its ω-limit set is defined by

$$(1.2)\qquad \omega(u)=\bigcap_{0<t<s(u)} \text{closure}\ \{u(\cdot,\tau):\tau>t\},$$

where the "closure" is with respect to the topology of $C^2(\boldsymbol{R})$.

A standard *a priori* estimate and dynamical systems argument show that $\omega(u)\neq\varnothing$ if and only if $s(u)=\infty$ and there exists a sequence $t_n\to\infty$ such that $u(\cdot,t_n)$ remains bounded as $n\to\infty$, in which case $\omega(u)$ is a connected locally compact subset of $C^2(\boldsymbol{R})$. Using the Lyapunov functional, one can show that each $\phi\in\omega(u)$ is an equilibrium of (1.1), that is, ϕ satisfies

$$(1.3)\qquad \begin{cases} \phi''+f(\phi)=0, & x\in\boldsymbol{R},\\ \phi(x+1)=\phi(x), & x\in\boldsymbol{R}. \end{cases}$$

For the details, see, for instance, [1] and [4].

For each $\lambda\in\boldsymbol{R}$, we let $\phi(x;\lambda)$ be the solution of the following initial value problem for ordinary differential equation:

$$(1.4)\qquad \begin{cases} \phi''+f(\phi)=0, & x\in\boldsymbol{R},\\ \phi(0;\lambda)=\lambda,\\ \phi'(0;\lambda)=0, \end{cases}$$

where $'$ stands for d/dx. Define

$$\Lambda(f)=\{\lambda\in\boldsymbol{R}:\phi(x;\lambda)\text{ is nonconstant and 1-periodic in } x\}.$$

Our main result is the following:

Theorem. *If $\Lambda(f)$ contains no interior point, then $\omega(u)$ contains at*