93. *A Remark on the Essential Self-adjointness of Dirac Operators*

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In this paper we shall consider the essential self-adjointness of Dirac operators

$$H=\sum_{j=1}^{3}\alpha_j D_j+\beta+Q(x),\quad x\in \boldsymbol{R}^3,\quad D_j=\frac{1}{i}\frac{\partial}{\partial x_j}$$

defined on $[C_0^\infty(\boldsymbol{R}^3)]^4$, where α_j and $\alpha_4=\beta$ are 4×4 Hermitian symmetric matrices satisfying

$$\alpha_j^2=I,\quad \alpha_j\alpha_k+\alpha_k\alpha_j=0\quad (j\neq k)$$

(I is the unit matrix). We define α_r by

$$\alpha_r=\sum_{j=1}^{3}(x_j/r)\alpha_j \qquad (r=|x|)$$

which is Hermitian symmetric for each $x\neq 0$ and satisfies

$$\alpha_r^2=I \tag{1}$$

in view of the above anti-symmetric relations. The potential $Q(x)$ is a 4×4 Hermitian symmetric matrix valued function of the following form

$$Q(x)=\frac{ib_1}{r}\alpha_r\beta+\frac{b_2}{r}\beta+V(x),$$

where b_1, b_2 are real constants. M. Arai [1], Theorem 3.1, shows that H is essentially self-adjoint and that the domain of the closure $\bar{H}$ coincides with the Sobolev space $[H^1(\boldsymbol{R}^3)]^4$, if

$$r\left|V(x)+\frac{i}{2r}\alpha_r\right|\leqq m \tag{2}$$

for a positive constant m such that

$$m<m_0\equiv \min_{k\in \boldsymbol{Z}\setminus\{0\}}\sqrt{(k+b_1)^2+b_2^2} \tag{3}$$

(see our Remark 8), where $|A|$ for a matrix A denotes the square root of the largest eigenvalue of A^*A. Moreover, Arai [1], Theorem 2.7, proves for the Coulomb potential $V(x)=(e/r)I$ that H is essentially self-adjoint if and only if $e^2\leqq m_0^2-(1/4)$.

Our result is that we can take $m=m_0$ in (2), that is,

Theorem 1. *If the potential $Q(x)$ satisfies*

$$r\left|V(x)+\frac{i}{2r}\alpha_r\right|\leqq m_0, \tag{4}$$

then H is essentially self-adjoint.

Corollary 2. *Let $m_0\geqq(1/2)$.*

(i) *If $V(x)$ satisfies*