86. *Products of Compact Fréchet Spaces*

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(Communicated by Kôsaku YOSIDA, M. J. A., Oct. 13, 1986)

1. Introduction. A topological space X is called *Fréchet* if each point in the closure of a subset $A \subset X$ is the limit of a sequence from A. In 1972, E. Michael [5] raised the question whether there exist two compact Hausdorff spaces X and Y such that the product space $X \times Y$ is not Fréchet. After then, various examples of such spaces were constructed by T. K. Boehme and M. Rosenfeld [1] (under the continuum hypothesis CH), V. I. Malyhin and B. E. Šapirovskiĭ [4], R. C. Olson [7] (under Martin's axiom), and P. Simon [8] (without extra set-theoretic assumptions).

Generalizing E. Michael's question for more than two compact spaces, T. Nogura [6] asked: For $n \geqq 2$, is there a compact Fréchet space X such that X^n is Fréchet but X^{n+1} is not Fréchet?

The purpose of this paper is to answer the question positively under Martin's axiom. Indeed, using G. Gruenhage's technique [2] and Franklin compact spaces, we construct for each n with $3 \leqq n \leqq \omega$, a compact Fréchet space X such that X^k is Fréchet for any $k < n$, but X^n is not Fréchet.

All spaces are assumed to be Hausdorff. The symbol ω denotes the first infinite cardinal and, simultaneously, the set of non-negative integers with the discrete topology.

2. Preliminaries. The Stone-Čech compactification of the countable discrete space ω is denoted by $\beta\omega$. For each $A \subset \omega$, the set A^* is defined by $A^* = \mathrm{cl}_{\beta\omega} A - A$. Let $\mathcal{P}$ be an infinite family of disjoint clopen subsets of ω^*. The *Franklin compact space* $F(\mathcal{P})$ is a quotient space of $\beta\omega$ obtained by the decomposition of $\beta\omega$ into $\{\omega^* - \bigcup \mathcal{P}\}$, elements of $\mathcal{P}$, and one-point sets $\{n\}$ with $n \in \omega$. Express $F(\mathcal{P})$ as $\{\infty\} \cup \mathcal{P} \cup \omega$, or more precisely, as $\{\infty_{\mathcal{P}}\} \cup \mathcal{P} \cup \omega$.

Note that every family $\mathcal{P}$ of disjoint clopen subsets of ω^* can be written as $\mathcal{P} = \{I^* : I \in \mathcal{I}\}$, where the family $\mathcal{I}$ is an almost disjoint family of infinite subsets of ω. A family $\mathcal{I}$ is said to be *almost disjoint* if $I \cap J$ is finite for any distinct members $I, J \in \mathcal{I}$.

It is easy to check the following lemma:

Lemma 1. (a) *$F(\mathcal{P})$ is a compact Hausdorff space.*

(b) *Each point of ω is isolated.*

(c) *Let $P \in \mathcal{P}$ and $P = I^*$, where $I \in \mathcal{I}$. Then the family $\{\{P\} \cup (I - F) :$ F is a finite subset of $\omega\}$ is a neighborhood base at the point $\{P\}$.*

(d) *The family $\{\{\infty_{\mathcal{P}}\} \cup (\mathcal{P} - Q) \cup (\omega - \bigcup Q - F) : Q$ is a finite subfamily of $\mathcal{P}$, and F is a finite subset of $\omega\}$ is a neighborhood base at the point $\infty_{\mathcal{P}}$.*