9. *The Number of Embeddings of Integral Quadratic Forms. II*[*]

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This is a continuation of our previous note [5], to which we refer the reader for definitions and notation.

1. Introduction. Let $\phi: M \to L$ be a primitive embedding from a nondegenerate integral quadratic form M into an indefinite unimodular integral quadratic form L. In [5] we showed that the number of equivalence classes of primitive embeddings from M into L coincides with a certain invariant $e(N)$ of the orthogonal complement N of M in L. (We also proved a similar statement for (α, β)-equivalence classes and the invariant $e_{\alpha\beta}(N)$.) In this note, we give an effective procedure for calculating these invariants $e(N)$ and $e_{\alpha\beta}(N)$ when N is indefinite with rank at least three. This extends some work of Nikulin [6], who gave sufficient conditions for $e(N)$ to be 1 (under the same hypotheses on N). The proofs, together with some applications to algebraic geometry, will be given elsewhere.

2. The structure of finite quadratic forms. A *finite quadratic form* is a finite abelian group G together with a map $q: G \to \boldsymbol{Q}/\boldsymbol{Z}$ such that the induced map $b: G \times G \to \boldsymbol{Q}/\boldsymbol{Z}$ defined by $b(x, y) = q(x+y) - q(x) - q(y)$ is $\boldsymbol{Z}$-bilinear, and such that $q(nx) = n^2 q(x)$ for all $n \in \boldsymbol{Z}$ and $x \in G$. G is called *nondegenerate* if the adjoint map $\mathrm{Ad}\, b: G \to \mathrm{Hom}(G, \boldsymbol{Q}/\boldsymbol{Z})$ of the associated bilinear form b is injective.

We recall from Wall [8] and Durfee [2] the basic structure of a nondegenerate finite quadratic form G, using the notation of Brieskorn [1]. The Sylow decomposition $G = \oplus_p G_p$ is an orthogonal direct sum decomposition with respect to the form q; moreover, each Sylow subgroup G_p admits an orthogonal direct sum decomposition into groups of ranks one and two of the following types:

(i) If $p \neq 2$ and $\varepsilon = \pm 1$, $w^{\varepsilon}_{p,k}$ denotes $\boldsymbol{Z}/p^k\boldsymbol{Z}$ with a generator x such that the quadratic map is given by $q(x) = p^{-k}u \pmod{\boldsymbol{Z}}$ for some $u \in \boldsymbol{Z}$ with $(u, p) = 1$ and $\left(\frac{2u}{p}\right) = \varepsilon$, where $\left(\frac{\ }{\ }\right)$ is the Legendre symbol.

(ii) If $\varepsilon \in (\boldsymbol{Z}/8\boldsymbol{Z})^{\times}$, $w^{\varepsilon}_{2,k}$ denotes $\boldsymbol{Z}/2^k\boldsymbol{Z}$ with a generator x such that $q(x) = 2^{-k-1}u \pmod{\boldsymbol{Z}}$ for some $u \in \boldsymbol{Z}$ with $u \equiv \varepsilon \pmod 8$.

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