80. *Large-time Behavior of Solutions for Hyperbolic-parabolic Systems of Conservation Laws*

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1. Introduction. We consider the system of conservation equations

(1) $$f^0(u)_t+f(u)_x=(G(u)u_x)_x, \quad t\geq 0, \quad x\in \boldsymbol{R},$$

where $u=u(t,x)$ is an m-vector, $f^0(u)$ and $f(u)$ are smooth m-vector valued functions, and $G(u)$ is a smooth $m\times m$ matrix. We assume that $Df^0(u)$, the Jacobian of $f^0(u)$, is non-singular and the mapping $v=f^0(u)$ is one-to-one so that (1) is equivalent to

(2) $$v_t+f(u(v))_x=(\tilde{B}(u(v))v_x)_x, \qquad v=f^0(u).$$

Here $u=u(v)$ is the inverse mapping of $v=f^0(u)$ and $\tilde{B}(u)=G(u)Df^0(u)^{-1}$. We study the large-time behavior of solution of (1). It is shown that as $t\to\infty$, the solution of (1) approaches the superposition of the nonlinear and linear diffusion waves constructed by solutions of the Burgers equation and the linear heat equation. The same problem was discussed in [5] for a model system of a viscous gas.

2. Global existence and decay of solutions. As the first step, we consider the global existence problem for (1). This problem has been solved in [2] under the conditions (i)–(iii) described below.

(i) The system (1) has a strictly convex entropy ([1], [2]).

This condition enables us to reduce the system (1) to the symmetric form

(3) $$A^0(u)u_t+A(u)u_x=B(u)u_{xx}+g(u,u_x).$$

Here $A^0(u)$, $A(u)$ and $B(u)$ are $m\times m$ symmetric matrices such that $A^0(u)$ is positive definite and $B(u)$ is nonnegative definite. For the explicit form of (3), see [1], [2].

(ii) The associated symmetric system (3) is hyperbolic-parabolic ([2]).

(iii) The linearized system of (3) around a given constant state $u=\bar{u}$ satisfies the stability condition ([6]): Let $\lambda A^0(\bar{u})\phi=A(\bar{u})\phi$ and $B(\bar{u})\phi=0$ for $\lambda\in\boldsymbol{R}$ and $\phi\in\boldsymbol{R}^m$. Then $\phi=0$.

The results concerning the global existence and decay of solution of (1) are summarized in the following theorem.

Theorem 1 ([2]). *Let $\bar{u}$ be a constant state and assume* (i)–(iii). *Consider* (1) *with the initial data $u(0,x)=u_0(x)$. If $u_0(x)-\bar{u}$ is small in H^s, $s\geq 2$, then* (1) *has a unique global solution $u(t,x)$ which converges to $\bar{u}$ uniformly in $x\in\boldsymbol{R}$ as $t\to\infty$. If, in addition, $u_0(x)-\bar{u}$ is small in $H^s\cap L^1$, $s\geq 3$, then the L^2-norm of $\partial_x^l(f^0(u(t,x))-f^0(\bar{u}))$ tends to zero at the rate $t^{-(1/2+l)/2}$ as $t\to\infty$, where $3l\leq s-2$.*

The first part of the theorem is proved by an energy method which