

79. The n -Plane with Integral Curvature Zero and without Conjugate Points

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0. Introduction. Cohn-Vossen ([2]) proved that a plane without conjugate points has the nonpositive integral Gaussian curvature if it exists ([1]). Recently, Green-Gulliver ([4]) has proved that a plane whose metric differs from the canonical flat metric at most on a compact set is Euclidean if there is no conjugate point. The assumption implies automatically that its integral Gaussian curvature is zero. The purpose of the present note is to show the following

Theorem. *Let M be an n -plane without conjugate points. If the absolute integral Ricci curvature exists and the integral scalar curvature is zero, then M is Euclidean.*

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1. Preliminaries. Let M be a complete simply connected Riemannian manifold without conjugate points. Then M is diffeomorphic to the Euclidean space E^n , $n = \dim M$, and all geodesics are minimizing.

The limit solution of matrix Jacobi equation. Let $\gamma : (-\infty, \infty) \rightarrow M$ be a geodesic. Let R be the curvature tensor of M . We consider the $(n-1) \times (n-1)$ matrix Jacobi equation

$$(*) \quad D''(t) + R(t)D(t) = 0,$$

where $R(t) : \dot{\gamma}(t)^\perp \rightarrow \dot{\gamma}(t)^\perp$ is given by $R(t)(x) = R(x, \dot{\gamma}(t))\dot{\gamma}(t)$ for any $x \in \dot{\gamma}(t)^\perp$. It follows from [3] that if $D_s(t)$, $s > 0$, is the solution of (*) with $D_s(0) = I$, $D_s(s) = 0$, then $D(t) = \lim_{s \rightarrow \infty} D_s(t)$ exists for any $t \in (-\infty, \infty)$ and is a solution of (*) such that $D(0) = I$, $\det(D(t)) \neq 0$ for any $t \in (-\infty, \infty)$. Set $A(t) = D'(t)D^{-1}(t)$ for any $t \in (-\infty, \infty)$. Then it follows that $A(t)$ is symmetric and

$$A'(t) + A(t)^2 + R(t) = 0$$

for any $t \in (-\infty, \infty)$.

The volume form of the unit tangent bundles. Let SM be the unit tangent bundle. Let $f^t : SM \rightarrow SM$ be the geodesic flow, i.e., $f^t v = \dot{\gamma}_v(t)$, where $\gamma_v : (-\infty, \infty) \rightarrow M$ is the geodesic with $\dot{\gamma}(0) = v \in SM$. It is well-known that $d\omega = d\sigma \wedge d\theta$ is f^t -invariant, where $d\sigma$ is the volume form induced from the Riemannian metric and $d\theta$ is canonical volume form on the unit sphere in E^n , $n = \dim M$. We introduce an equivalence relation \sim in such a way that $v \sim w$ if $v = f^t w$ for some $t \in (-\infty, \infty)$, where $v, w \in SM$. Let N be the set of all equivalence classes $[v]$, $v \in SM$. All orbits $[v]$, $v \in SM$, are images