75. *A Problem of Orlicz in the Scottish Book*

By Minoru AKITA,*) Kazuo GOTO,**) and Takeshi KANO*)

(Communicated by Shokichi IYANAGA, M. J. A., Sept. 12, 1986)

Orlicz posed the following problem in the Scottish Book ([1], [2]).

Orlicz's Problem (No. 121). *Give an example of a trigonometric series*

$$\sum (a_n \cos nx + b_n \sin nx) \tag{1}$$

everywhere divergent and such that

$$\sum (|a_n|^{2+\varepsilon} + |b_n|^{2+\varepsilon}) < \infty \tag{2}$$

for every $\varepsilon > 0$.

Our main purpose is to give an example in answer to this problem.

The problem was studied by Banach and his colleagues, especially by Orlicz ([3]). It should be worth noticing that the real difficulty or spirit of the problem lies in the point that Orlicz asked a concrete example instead of a mere existence proposition, and also in the point that the example must furnish the property of everywhere divergence.

First of all, we shall mention some partial answers applying known results on the size of certain partial sums of the exponential series

$$\sum c_n e^{2\pi i n x}. \tag{3}$$

Proposition 1. *Set* $c_n = n^{-1/2} \exp(ian \log n)$ $(a > 0)$. *Then* (3) *diverges for almost all* x, *whereas*

$$\sum |c_n|^{2+\varepsilon} < \infty$$

for every $\varepsilon > 0$.

This follows from M. Weiss' result ([4]). It seems that she was not aware of Orlicz's problem. By using probability method, she investigated the behavior of the exponential series

$$\sum n^{-1/2} \exp(i\beta n \log n + in\theta).$$

Proposition 2. *Set*

$$c_n = \begin{cases} k^{-1/2}(\log k)^{-1/4} & \text{if } n = k^2 > 1, \\ 0 & \text{otherwise.} \end{cases}$$

Then holds the same conclusion as in Proposition 1.

This follows from Fiedler-Jurkat-Körner's result ([5]). They were interested in the almost all estimates of the sum

$$S_N(x) = \sum_{n \leq N} e^{\pi i n^2 x} \quad (x \in \boldsymbol{R}).$$

Proposition 3. *There exists a sequence* $c_n \to 0$ *such that* (3) *diverges for all* x, *yet*

$$\sum |c_n|^{2+\varepsilon} < \infty$$

for every $\varepsilon > 0$.

*) Department of Mathematics, Okayama University.

**) Sakuyo Junior College.