72. *Local Isometric Embedding Problem of Riemannian 3-manifold into* $\boldsymbol{R}^6$

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§ 1. **Introduction.** Although the problem of the existence of a local C^∞ isometric embedding for a Riemannian n-manifold (M, g) into Euclidean space $\boldsymbol{R}^{n(n+1)/2}$ is an old and famous problem, there are only a few results if $n \geqq 3$. Recently, Bryant-Griffiths-Yang [1] made a big contribution to the case $n=3$. In this paper, we generalize their results as follows:

Theorem 1. *Let (M, g) be a C^∞ Riemannian 3-manifold and $p_0 \in M$ be a point such that the curvature tensor $\boldsymbol{R}(p_0)$ does not vanish. Then there exists a local C^∞ isometric embedding of a neighborhood U_0 of p_0 into $\boldsymbol{R}^6$.*

The result of [1] treats under the additional assumption:

(*) $R(p_0)$ does not have signature $(0, 1)$,

where the signature of $R(p)$ is defined by considering $R(p)$ as a symmetric linear operator acting on the space of 2-forms.

§ 2. **Linearized PDE for the isometric embedding equation.** We shall consider the linearized PDE corresponding to the isometric embedding equation. Take $p_0 \in M$ as the origin and let $U(u^1, u^2, u^3)$ be a coordinate neighborhood around p_0. Let $(x^A(u))$ be a local C^∞ embedding of U into $\boldsymbol{R}^6$ and consider the following PDE for the unknown functions $(y^A(u))$:

$$(1)\qquad \nabla_i y_j + \nabla_j y_i = 2\sum_{\lambda=4}^{6} y_\lambda H_{ij\lambda}(u) + k_{ij}(u) \qquad i, j = 1, 2, 3,$$

where $(k_{ij}(u))$ is a symmetric 3×3 matrix depending smoothly on u. Here, choosing a unit normal frame field $\{N_\lambda(u)\}_{\lambda=4,5,6}$ on U, we set

$$y^A(u) = \sum_{i=1}^{6} y_i \frac{\partial x^A}{\partial u^i} + \sum_{\lambda=4}^{6} y_\lambda \cdot N_\lambda^A,$$

and denote by ∇ and $H_{ij\lambda}(u)$ the covariant derivatives and the second fundamental form in terms of the isometric embedding $(x^A)_{A=1,\dots,6}$ and the unit normal frame $\{N_\lambda\}$, respectively.

Definition 2. An isometric embedding is called *non-degenerate* if the corresponding second fundamental form $(H_{ij\lambda}(u))$ is linearly independent in the space of all 3×3 symmetric matrices at each point of U.

For a positive integer N, let P be an $N\times N$ system of classical pseudo-differential operator on M with the principal symbol $p(x, \xi)$.

Definition 3. P is called *a system of (real) principal type* at $x_0 \in M$ if, for any $(x_0, \xi_0) \in T^*M - \{0\}$, there exists a conic neighborhood Γ of (x_0, ξ_0), an $N\times N$ homogeneous classical symbol $\tilde{p}(x, \xi)$, and a (real valued) homo-

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