# 70. *Asymptotic Behavior of Solutions for the Equations of a Viscous Heat-conductive Gas*

By Shuichi KAWASHIMA,*) Akitaka MATSUMURA,**) and Kenji NISHIHARA***)

(Communicated by Kôsaku YOSIDA, M. J. A., Sept. 12, 1986)

**1. Introduction.** We study the asymptotic behavior of solutions to the initial value problem for the equations of a viscous heat-conductive gas in Lagrangian coordinates:

$$(1) \qquad \begin{aligned} &v_t - u_x = 0, \qquad u_t + p_x = (\mu u_x / v)_x, \\ &(e + u^2/2)_t + (pu)_x = (\kappa \theta_x / v + \mu u u_x / v)_x, \end{aligned}$$

where the unknowns $v>0$, $u$ and $\theta>0$ represent the specific volume, the velocity and the absolute temperature of the gas. The coefficients of viscosity and heat-conductivity, $\mu$ and $\kappa$, are assumed to be positive constants. The pressure $p$, the internal energy $e$ and the entropy $s$ are smooth functions of $(v, \theta)$. Also, $p$ and $e$ are regarded as smooth functions of $(v, s)$. We write $p=p(v,\theta)=\hat{p}(v,s)$, $e=e(v,\theta)=\hat{e}(v,s)$, $s=s(v,\theta)$ and assume that $\partial p(v,\theta)/\partial v<0$, $\partial e(v,\theta)/\partial\theta>0$, $\partial^2\hat{p}(v,s)/\partial v^2>0$ and $\hat{p}(v,s)$ is a convex function of $(v,s)$. These conditions together with the thermodynamic relation $de=\theta ds-pdv$ ensure that the corresponding inviscid system

$$(2) \qquad v_t - u_x = 0, \quad u_t + p_x = 0, \quad (e+u^2/2)_t + (pu)_x = 0$$

is strictly hyperbolic and each characteristic field is either genuinely nonlinear or linearly degenerate ([2]).

We denote the initial function for (1) by $U_0(x)=(v_0, u_0, \theta_0)(x)$ and put $U_\pm = U_0(\pm\infty)$. When $U_- = U_+$, it was shown in [6] that the solution of (1) converges to the constant state $U_- = U_+$ as $t\to\infty$. The case $U_- \neq U_+$ was studied recently in [4], [1], [3] under the hypothesis that $U_-$ is connected to $U_+$ by only shock waves. It was proved that the solution of (1) approaches the superposition of smooth traveling waves with shock profile. In this paper, we consider the case where $U_-$ is connected to $U_+$ by only rarefaction waves, and show that the solution of (1) converges to the weak solution of the Riemann problem for the inviscid equations (2). A similar result has been obtained in [5] for the barotropic model gas.

**2. Theorems.** In what follows, we assume that $\delta=|U_+ - U_-|$ is small and $U_-$ is connected to $U_+$ by only rarefaction waves. We denote by $\bar{U}(t,x)=(\bar{v}, \bar{u}, \bar{\theta})(t,x)$ the weak solution to the Riemann problem for (2) with the step initial data $\bar{U}_0(x)=(\bar{v}_0, \bar{u}_0, \bar{\theta}_0)(x)=U_\pm$, $x\gtrless 0$ (cf. [2]). Our main result is stated as follows.

---

*) Department of Mathematics, Nara Women's University.
**) Department of Applied Mathematics and Physics, Kyoto University.
***) Tokyo National Technical College, Hachioji, Tokyo.