69. *Fluctuation of Spectra in Random Media*

By Shin OZAWA
Department of Mathematics, Osaka University

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§1. **Introduction.** The Lenz shift phenomena were studied by various authors by various methods. See Kac [5] Huruslov-Marchenko [4], Rauch-Taylor [13], Papanicolaou-Varadhan [14], Ozawa [8], [9], [10], [12], Chavel-Feldman [1]. In [3], Figari-Orlandi-Teta gave fluctuation result for the Lenz shift phenomena by developing the method of [8]. In [8], perturbative calculus using the Green function was offered. It turned out to be strong enough to consider fluctuation of spectra.

In the present note we give Theorem 1 on the Lenz shift.

We consider a bounded domain Ω in $\boldsymbol{R}^3$ with smooth boundary γ. We put $B(\varepsilon; w)=\{x\in \boldsymbol{R}^3; |x-w|<\varepsilon\}$. Fix $\beta\geq 1$. Let

$$0<\mu_1(\varepsilon; w(m))\leq \mu_2(\varepsilon; w(m))\leq \cdots$$

be the eigenvalues of $-\Delta\,(=-\mathrm{div}\ \mathrm{grad})$ in the set $\Omega_{\varepsilon,w(m)}=\Omega\backslash\overline{\bigcup_{i=1}^{\tilde m}B(\varepsilon; w_i^{(m)})}$ under the Dirichlet condition on its boundary. Here $\tilde m$ denotes the largest integer which does not exceed m^β and $w(m)$ denotes the set of $\tilde m$-points $\{w_i^{(m)}\}_{i=1}^{\tilde m}\in\Omega^{\tilde m}$. Let $V(x)>0$ be C^0-class function on Ω satisfying

$$\int_\Omega V(x)dx=1.$$

We consider Ω as the probability space with a probability density $V(x)$. Let $\Omega^{\tilde m}=\prod_{i=1}^{\tilde m}\Omega$ be the probability space with the product measure. Fix $\alpha>0$. Then, $\mu_j(\alpha/m; w(m))$ is a random variable on $\Omega^{\tilde m}$. Our aim is to know a precise asymptotic behaviour of $\mu_j(\alpha/m; w(m))$ as $m\to\infty$.

Main result is the following:

Theorem 1. *Fix j. Assume $\beta\in[1, 12/11)$, $V(x)=1/|\Omega|$ (const.). Assume that the j-th eigenvalue μ_j of the Laplacian in Ω under the Dirichlet condition on γ is simple. Then, the random variable*

$$m^{1-(\beta/2)}((\mu_j(\alpha/m; w(m))-(\mu_j+4\pi\alpha m^{\beta-1}|\Omega|^{-1}))) \tag{1}$$

tends in distribution to Gaussian random variable Π_j of mean $E(\Pi_j)=0$ and variance

$$E(\Pi_j^2)=4\pi\alpha\left(\int_\Omega \varphi_j(x)^4|\Omega|^{-1}dx-\left(\int_\Omega \varphi_j(x)^2|\Omega|^{-1}dx\right)^2\right)$$

as m tends to ∞. Here φ_j is the normalized eigenfunction associated with μ_j.

Remark. Figari-Orlandi-Teta's result (see [3]) is the case $\beta=1$. As a corollary of Theorem 1, we have

$$\lim_{m\to\infty} \boldsymbol{P}(w(m)\in\Omega^{\tilde m}; (1)\leq c)=\frac{1}{\sqrt{2\pi v}}\int_{-\infty}^{c} e^{-t^2/2v}dt,$$

where $v=E(\Pi_j^2)$.