65. *Characterizations of P^3 and Hyperquadrics Q^3 in P^4*

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Introduction. A compact complex threefold is called a Moishezon threefold if it has three algebraically independent meromorphic functions on it. The main consequences we report are

(0.1) **Theorem.** *Let X be a compact complex threefold or a complete irreducible nonsingular algebraic threefold defined over an algebraically closed field of arbitrary characteristic. Assume* $\mathrm{Pic}\, X = \boldsymbol{Z}L$, $H^1(X, \mathcal{O}_X)=0$, $K_X = -dL$, $d \geqq 4$ *(resp.* $d=3$*),* $L^3>0$, *and that* $h^0(X, mL) \geqq 2$ *for some positive integer* m. *Then X is isomorphic to projective space $\boldsymbol{P}^3$ (resp. a nonsingular hyperquadric $\boldsymbol{Q}^3$ in $\boldsymbol{P}^4$).*

(0.2) **Theorem.** *A compact complex threefold homeomorphic to $\boldsymbol{P}^3$ (resp. $\boldsymbol{Q}^3$) is isomorphic to $\boldsymbol{P}^3$ (resp. $\boldsymbol{Q}^3$) if* $H^q(X, \mathcal{O}_X)=0$ *for any* $q>0$ *and if* $h^0(X, -mK_X) \geqq 2$ *for some positive integer* m.

(0.3) **Theorem.** *A Moishezon threefold homeomorphic to $\boldsymbol{P}^3$ (resp. $\boldsymbol{Q}^3$) is isomorphic to $\boldsymbol{P}^3$ (resp. $\boldsymbol{Q}^3$) if its Kodaira dimension is less than three.*

(0.4) **Theorem.** *An arbitrary complex analytic* (*global*) *deformation of $\boldsymbol{P}^3$ (resp. $\boldsymbol{Q}^3$) is isomorphic to $\boldsymbol{P}^3$ (resp. $\boldsymbol{Q}^3$).*

The theorems (0.2)–(0.4) are derived from (0.1), see section 3. The theorems (0.2)–(0.4) in arbitrary dimension have been proved by Hirzebruch-Kodaira [3] and Yau [14] (resp. by Brieskorn [1]) under the assumption that the manifold is Kählerian. See [2], [5], [6] for related results. Recently Tsuji [12] claims that he is able to prove the theorem (0.4) for $\boldsymbol{P}^n$, whereas Peternell [9] asserts the theorems (0.3) and (0.4) in a stronger form. However there is a gap in the proof of [9], as Peternell himself admits at the end of the article. After the author completed [7] and the major parts of [8], he received two preprints of Peternell [10], [11] in which Peternell completes the proof in [9] of the theorems (0.3) and (0.4) assuming no conditions on Kodaira dimension.

In [7], [8], we make an approach different from theirs and give an elementary proof of the above theorems. Our idea of the proof of (0.1) is as follows. First we see $h^0(X, L)>2$ and then take two distinct members D, D' of the linear system $|L|$. We determine all the possible structures of the scheme-theoretic complete intersection $l = D \cap D'$. From this we easily see that $L^3=1$ (resp. 2), $h^0(X, L)=4$ (resp. 5), and that $|L|$ is base point free. Moreover we see that the morphism associated with $|L|$ is an isomorphism of X onto $\boldsymbol{P}^3$ (resp. $\boldsymbol{Q}^3$).

§1. Proof of (0.1)—the case of projective space $\boldsymbol{P}^3$. In this section