61. *Class Number Relations of Algebraic Tori. I*

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Let k be an algebraic number field of finite degree and $\mathfrak{p}$ be a place of k. We denote by $k_{\mathfrak{p}}$ the completion of k at the place $\mathfrak{p}$. $O_{\mathfrak{p}}$ denotes the ring of $\mathfrak{p}$-adic integers when $\mathfrak{p}$ is non-archimedean, and $k_{\mathfrak{p}}$ when $\mathfrak{p}$ is archimedean. Thus $U_k=\prod_{\mathfrak{p}} O_{\mathfrak{p}}^{\times}$ is a subgroup of the idele group $k_A^{\times}$. Let T be a torus defined over k and $\hat{T}=\mathrm{Hom}\,(T, G_m)$ be the character module of T. We denote by $T(k)$ the group of k-rational points of T, and by $T(k_{\mathfrak{p}})$ the group of $k_{\mathfrak{p}}$-rational points of T. $T(O_{\mathfrak{p}})$ denotes the unique maximal compact subgroup of $T(k_{\mathfrak{p}})$ when $\mathfrak{p}$ is non-archimedean, and $T(k_{\mathfrak{p}})$ when $\mathfrak{p}$ is archimedean. We put $T(U_k)=\prod_{\mathfrak{p}} T(O_{\mathfrak{p}})$, $T(O_k)=T(U_k)\cap T(k)$ and denote the adele group of T over k by $T(k_A)$. Then $T(U_k)$ is a subgroup of $T(k_A)$. The class number of T over k is defined by

$$h(T)=[T(k_A) : T(k)\cdot T(U_k)].$$

Consider the exact sequence of algebraic tori defined over k

$$(1) \qquad 0 \longrightarrow T' \xrightarrow{\alpha} T \xrightarrow{\mu} T'' \longrightarrow 0,$$

where α and μ are defined over k.

Recently, T. Ono treated the case when $T=R_{K/k}(G_m)$ and $T''=G_m$ in (1), where K is a finite Galois extension of k and $R_{K/k}$ is the Weil map. In his paper [3], he defined the number $E(K/k)$ by $h(R_{K/k}(G_m))/h(T')\cdot h(G_m)$ and obtained an equality between $E(K/k)$ and some elementary cohomological invariants of K/k in [4], [5].

In this paper, we shall obtain a similar equality between $h(T)/h(T')\cdot h(T'')$ and some cohomological invariants. Moreover, we shall define a number $E'(K/k)$ for any finite Galois extension K/k and investigate the relation between $E(K/k)$ and $E'(K/k)$.

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Let A, B be commutative groups and λ be a homomorphism from A to B. If $\mathrm{Ker}\,\lambda$, $\mathrm{Cok}\,\lambda$ are finite, we define the q-symbol of λ by $q(\lambda)=[\mathrm{Cok}\,\lambda]/[\mathrm{Ker}\,\lambda]$. Let $\lambda: T\to T'$ be a k-isogeny of algebraic tori. Then λ induces the following natural homomorphisms

$$\hat{\lambda}(k) : \hat{T}'(k) \longrightarrow \hat{T}(k),$$
$$\lambda(O_{\mathfrak{p}}) : T(O_{\mathfrak{p}}) \longrightarrow T'(O_{\mathfrak{p}}),$$
$$\lambda(O_k) : T(O_k) \longrightarrow T'(O_k).$$

Here $\hat{T}(k)$ denotes the submodule of $\hat{T}$ consisting of rational characters defined over k. Then one knows