7. *Simple Vector Bundles over Symplectic Kähler Manifolds*

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1. Introduction. In a recent paper [5], Mukai has shown that the moduli space of simple sheaves on an abelian or K3 surface is smooth and has a holomorphic symplectic structure. We extend his result to higher dimensional manifolds by a differential geometric method.

A *holomorphic symplectic structure* on a complex manifold is given by a closed holomorphic 2-form ω which is non-degenerate in the sense that if $\omega(u, v)=0$ for all tangent vectors v, then $u=0$.

Let M be a compact Kähler manifold of dimension n and E a C^∞ complex vector bundle of rank r over M. Let $A^{p,q}(E)$ be the space of C^∞ (p, q)-forms over M with values in E. A *semi-connection* in E is a linear map $D'' : A^{0,0}(E) \to A^{0,1}(E)$ such that

(1.1) $$D''(as)=d''a \cdot s + aD''s$$

for all functions a on M and all sections s of E. Let $\mathcal{D}''(E)$ denote the space of semi-connections in E. Every semi-connection D'' extends uniquely to a linear map $D'' : A^{p,q}(E) \to A^{p,q+1}(E)$ such that

(1.2) $$D''(\alpha \wedge \sigma) = d''\alpha \wedge \sigma + (-1)^r \alpha \wedge D''\sigma$$

for all r-forms α on M and all $\sigma \in A^{p,q}(E)$. In particular,

(1.3) $$N(D'') := D'' \circ D'' : A^{0,0}(E) \longrightarrow A^{0,2}(E),$$

and $N(D'')$ may be considered as an element of $A^{0,2}(\mathrm{End}\,(E))$. A semi-connection D'' is called a *holomorphic structure* if $N(D'')=0$. Let $\mathcal{H}''(E)$ denote the set of holomorphic structures in E. If E is holomorphic, then $d'' \in \mathcal{H}''(E)$. Conversely, every $D'' \in \mathcal{H}''(E)$ comes from a unique holomorphic structure in E. The holomorphic vector bundle defined by D'' is denoted by $E^{D''}$. We call $E^{D''}$ *simple* if its endomorphisms are all of the form cI_E, where $c \in \boldsymbol{C}$. Let

(1.4) $$\mathrm{End}^0\,(E^{D''}) = \{u \in \mathrm{End}\,(E^{D''})\,;\ \mathrm{Tr}\,(u)=0\}.$$

Then $E^{D''}$ is simple if and only if $H^0(M, \mathrm{End}^0\,(E^{D''}))=0$. Let $\mathcal{S}''(E)$ denote the set of simple holomorphic structures D'' in E.

Let $GL(E)$ be the group of C^∞ automorphisms of the bundle E. Its Lie algebra $\mathfrak{gl}(E)$ is nothing but $A^{0,0}(\mathrm{End}\,(E))$. The group $GL(E)$ acts on $\mathcal{D}''(E)$ by

(1.5) $$D''^f = f^{-1} \circ D'' \circ f \qquad \text{for } f \in GL(E),\ D'' \in \mathcal{D}''(E).$$

Then $GL(E)$ leaves $\mathcal{H}''(E)$ and $\mathcal{S}''(E)$ invariant. With the C^∞ topology, the *moduli space* $\mathcal{S}''(E)/GL(E)$ *of simple holomorphic* structures in E is a (possibly non-Hausdorff) complex analytic space. As was shown by Kim

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