6. $\bar{\partial}$-Problem on a Family of Weakly Pseudoconvex Manifolds

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(Communicated by Kôsaku YOSIDA, M. J. A., Jan. 13, 1986)

1. In the case of weakly pseudoconvex manifolds the $\bar{\partial}$-problem depends not only on boundary conditions, but also on complex structures ([2, 3, 4, 5, 7, 8]). In this paper we investigate the $\bar{\partial}$-problem for the Picard variety $\mathrm{Pic}^0(T^n)$ of a complex n-dimensional torus T^n, which is regarded as a family of weakly pseudoconvex manifolds. For this problem we find a criterion given by the theory of Diophantine approximation. Full details will be published elsewhere.

2. Let $E \in \mathrm{Pic}^0(T^n)$. Then E is a holomorphic line bundle on T^n with Chern class zero. By a result of [1] we find a proper weakly plurisubharmonic C^∞-function $\Phi : E \to [0, \infty)$. Thus, we can regard $\mathrm{Pic}^0(T^n)$ as a family of noncompact weakly pseudoconvex manifolds. Since $\mathrm{Pic}^0(T^n)$ is isomorphic to a complex n-dimensional torus $T^{n*}=C^n/\Lambda$, we can define on $\mathrm{Pic}^0(T^n)$ an invariant distance $d(E, F) := \min\{\|a-b+c\| ; E=a+\Lambda, F=b+\Lambda \in C^n/\Lambda, c \in \Lambda\}$, where $\|(z_1, \cdots, z_n)\| := \max|z_i|$.

Theorem. *Let $E \in \mathrm{Pic}^0(T^n)$ and O the structure sheaf of E. Then E must be one of the following types:*

(1) *If $d(1, E^l)=0$ for some $l \geqq 1$, then $H^p(E, O)$ is an infinite-dimensional Hausdorff space $(1 \leqq p \leqq n)$;*

(2) *If there exists $a>0$ such that $\exp(-al) \leqq d(1, E^l)$ for any $l \geqq 1$, then $\dim H^p(E, O) = \binom{n}{p}$ $(1 \leqq p \leqq n)$;*

(3) *If $d(1, E^l) \neq 0$ for any $l \geqq 1$ and $\liminf_{l\to\infty} \exp(al)d(1, E^l)=0$ for any $a>0$, then $H^p(E, O)$ is not Hausdorff $(1 \leqq p \leqq n)$.*

Further let P_1, P_2 and P_3 be the subsets of $\mathrm{Pic}^0(T^n)$ consisting of the elements of the above types (1), (2) and (3), respectively. Then P_i is nonempty $(i=1, 2, 3)$, $P_1 \cup P_3$ is of Lebesgue measure zero and

$$\mathrm{Pic}^0(T^n) = P_1 \cup P_2 \cup P_3 \quad \text{(disjoint)}.$$

Remark. In comparison with strongly pseudoconvex manifolds, this theorem shows that a strange phenomenon occurs for weak pseudoconvexity. We have $P_2 \cup P_3 = \{E \in \mathrm{Pic}^0(T^n) ; H^0(E, O)=C\}$. E contains T^n as its zero section. If $E \in P_2$, then $H^p(E, O) \cong H^p(T^n, O_{T^n})$. This is similar to the case of a strongly pseudoconvex manifold and its exceptional set. But if $E \in P_3$, there exists a great difference from strong pseudoconvexity.

Proof. Let $E \in \mathrm{Pic}^0(T^n)$. We have a bireal-analytic isomorphism T^n

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