53. *On Compactifiable Strongly Pseudoconvex Surfaces*

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Throughout, analytic surfaces will mean 2-dimensional C-analytic manifolds. Purely 1-dimensional C-analytic spaces will be referred to simply as analytic curves. Furthermore, all compact analytic surfaces are assumed to be *minimal* [7], i.e., free from exceptional curves of the first kind.

1. Structures of compactifiable strongly pseudoconvex surfaces.

Definition 1 [8], [9]. A non compact analytic surface X is said to be *strongly pseudoconvex* if i) X is holomorphically convex and if ii) there exists a compact analytic curve $E\subset X$ such that $T\subseteq E$ for any irreducible compact analytic curve $T\subset X$.

E is called the *exceptional* curve of X. In the special case where $E=\phi$, X is called a *Stein* surface.

Definition 2. Let X be a non compact analytic surface. A compact analytic surface M is said to be a *compactification* of X if there exists a C-analytic subvariety $\Gamma\subset M$ such that X is biholomorphic to $M\backslash\Gamma$. Furthermore, M is said to be an algebraic (or a non algebraic) compactification if M is an algebraic (or a non algebraic) surface. X is called *compactifiable* if it admits some compactification M.

Remark 1. If X is a strongly pseudoconvex or a Stein surface, then one can check that Γ is a compact connected analytic curve [3].

Our main goal here is to investigate the global structures of i) compactifiable Stein surfaces and ii) compactifiable strongly pseudoconvex surfaces which are not Stein (i.e. $E\neq\phi$).

Remark 2. In view of Definition 1, Stein surfaces are merely special cases of strongly pseudoconvex surfaces; so one might wonder why the treatment of those two surfaces has to be dealt with separately. One of our main purposes here is to point out the sharp contrast between those two cases from the view point of compactification. Hence, throughout, strongly pseudoconvex surfaces are meant to be not Stein!

Our investigation is motivated by the following results:

Theorem 1 [3], [11]. *Let M be a compactification of some Stein surface X. Then M is either* i) *an algebraic surface,* ii) $b_1=1$, $b_2=0$, *M admits no non constant meromorphic functions and contains exactly one compact analytic curve or* iii) $b_1=1$, $b_2>0$ *and M contains exactly one compact analytic curve.*