51. *Magnetohydrodynamic Approximation of the Complete Equations for an Electromagnetic Fluid. II*

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1. Introduction and equations. In the previous paper [2], we justified the magnetohydrodynamic approximation locally in time for certain two-dimensional flow of an electrically conducting compressible fluid. It was proved that the magnetohydrodynamic equations were obtained as the singular limit of the complete equations at the vanishing of the dielectric constant. The aim of this note is to justify the approximation globally in time in case where the fluid is viscous and heat-conductive.

The equations considered are

$$\begin{aligned}&\rho_t+\operatorname{div}(\rho u)=0,\\ &\rho(u_t+(u\cdot\nabla)u)+\nabla p=\operatorname{div}(2\mu P+\mu' I\operatorname{div}u)+J\times B,\\ (1)\qquad&\rho e_\theta(\theta_t+u\cdot\nabla\theta)+\theta p_\theta\operatorname{div}u=\operatorname{div}(\kappa\nabla\theta)+\Psi+J(E+u\times B),\\ &\varepsilon E_t-(1/\mu_0)\operatorname{rot}B+J=0,\\ &B_t+\operatorname{rot}E=0,\end{aligned}$$

$$(2)\qquad \operatorname{div}B=0.$$

Here and in the sequel, we use the notations for two-dimensional vectors. The unknowns $\rho>0$, $u=(u^1, u^2)$, $\theta>0$, E (scalar) and $B=(B^1, B^2)$ represent the mass density, the velocity, the absolute temperature, the electric field and the magnetic flux density, respectively. They are functions of time $t\geq 0$ and space variable $x=(x_1, x_2)\in\boldsymbol{R}^2$. The pressure p and the internal energy e are smooth functions of (ρ, θ) such that $p_\rho=\partial p/\partial\rho>0$ and $e_\theta=\partial e/\partial\theta>0$. The thermodynamic law $de=\theta dS-pd(1/\rho)$ is always assumed, where S (the entropy) is a smooth function of (ρ, θ). P is the deformation tensor, whose entries are $P_{ij}=(1/2)(\partial_j u^i+\partial_i u^j)$, $i, j=1, 2$, where $\partial_i=\partial/\partial x_i$.

$$\Psi=2\mu\sum P_{ij}^2+\mu'(\operatorname{div}u)^2$$

is the viscous dissipation function. The current density J (scalar) is given by Ohm's law $J=\sigma(E+u\times B)$. The viscosity coefficients μ and μ', the heat conductivity coefficient κ and the electric conductivity coefficient σ are smooth functions of (ρ, θ) such that $\mu>0$, $2\mu+\mu'>0$, $\kappa>0$ and $\sigma>0$. The dielectric constant ε and the magnetic permeability μ_0 are assumed to be positive constants.

The magnetohydrodynamic equations corresponding to (1), (2) are given by

$$\begin{aligned}&\rho_t+\operatorname{div}(\rho u)=0,\\ (3)\qquad&\rho(u_t+(u\cdot\nabla)u)+\nabla p-(1/\mu_0)\operatorname{rot}B\times B=\operatorname{div}(2\mu P+\mu' I\operatorname{div}u),\\ &\rho e_\theta(\theta_t+u\cdot\nabla\theta)+\theta p_\theta\operatorname{div}u=\operatorname{div}(\kappa\nabla\theta)+\Psi+(1/\sigma\mu_0^2)(\operatorname{rot}B)^2,\end{aligned}$$