50. *Initial-boundary Value Problem for Parabolic Equation in L^1*

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Let Ω be a not necessarily bounded domain in R^n locally regular of class C^{2m} and uniformly regular of class C^m in the sense of F. E. Browder [4]. We consider the following parabolic initial-boundary value problem

(1) $\partial u/\partial t+A(x,t,D)u=f(x,t), \quad x\in\Omega, \quad 0<t\leqq T,$

(2) $B_j(x,t,D)u=0, \quad j=1,\cdots,m/2, \quad x\in\partial\Omega, \quad 0<t\leqq T,$

(3) $u(x,0)=u_0(x), \qquad x\in\Omega,$

in $L^1(\Omega)$. Here for each $t\in[0,T]$

$$A(x,t,D)u=\sum_{|\alpha|\leqq m} a_\alpha(x,t)D^\alpha$$

is a strongly elliptic linear differential operator of order m and

$$B_j(x,t,D)=\sum_{|\beta|\leqq m_j} b_{j\beta}(x,t)D^\beta, \qquad j=1,\cdots,m/2,$$

is a normal set of linear differential operators on $\partial\Omega$ of order less than m. Similar problem was discussed in [3], [9], [10] for equations with coefficients independent of t. In [3] with the aid of the theory of dual semigroups H. Amann showed that the associated elliptic operator generates an analytic semigroup in $L^1(\Omega)$ in case $m=2$.

Concerning the coefficients of $A(x,t,D)$ and $B_j(x,t,D)$ we assume the following regularity conditions:

(i) $a_\alpha(x,t)$, $|\alpha|=m$, and their derivatives $\partial a_\alpha(x,t)/\partial t$ with respect to t are bounded and uniformly continuous in $\bar\Omega\times[0,T]$.

(ii) $a_\alpha(x,t)$, $|\alpha|<m$, and their derivatives with respect to t are bounded and measurable, and continuous in t uniformly in $\bar\Omega\times[0,T]$.

(iii) The coefficients of $B_j(x,t,D)$ are extended to $\bar\Omega\times[0,T]$ so that $(\partial/\partial x)^\gamma b_{j\beta}(x,t)$, $(\partial/\partial t)(\partial/\partial x)^\gamma b_{j\beta}(x,t)$, $|\beta|\leqq m_j$, $|\gamma|\leqq m-m_j$, $j=1,\cdots,m/2$, are bounded and uniformly continuous in $\bar\Omega\times[0,T]$.

(iv) The formally constructed adjoint boundary value problem $(A'(x,t,D)$, $\{B'_j(x,t,D)\}_{j=1}^{m/2},\Omega)$ satisfies (i), (ii), (iii).

For the well-posedness of the problem (1)–(3) we assume that for each fixed $t\in[0,T]$ and $\theta\in[\pi/2,3\pi/2]$

$$(-1)^{m/2}e^{i\theta}(\partial/\partial\tau)^m+A(x,t,D), \qquad \{B_j(x,t,D)\}_{j=1}^{m/2}$$

satisfies the complementing condition in the cylindrical domain $\Omega\times(-\infty,\infty)$ ([1], [2]).

The operator $A(t)$ is defined as follows:

The domain $D(A(t))$ is the totality of functions u satisfying

(i) $u\in W^{m-1,q}(\Omega)$ for each $q\in[1,n/(n-1))$,