5. On the Automorphism Groups of a Compact Bordered Riemann Surface of Genus Five

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§1. **Introduction.** Let R be a compact bordered Riemann surface of genus g with k boundary components. If $2g+k-1\geqq 2$, the automorphism group of R is a finite group. Then we put $N(g, k)$ to be the maximum order of automorphism groups of R where the maximum is taken over all R of genus g with k boundary components. It is well known that $N(g, k)$ is equal to the maximum order of automorphism groups of Riemann surfaces of genus g deleted k points, and that every automorphism group of R is isomorphic to that of a compact Riemann surface (Oikawa [6]). For every $k\geqq 0$, $N(0, k)$, $N(1, k)$, $N(2, k)$, $N(3, k)$ and $N(4, k)$ are determined by Heins [2], Oikawa [6], Tsuji [7], Tsuji [8] and Kato [4], respectively. In the present paper, we shall determine $N(5, k)$.

§2. **Notation.** Let S be a compact Riemann surface of genus $g\geqq 2$, G be a conformal automorphism group of S and N be the order of G. Let $S_0=S/G$ be the quotient surface with conformal structure induced from S through π, where π is the projection mapping from S onto S_0. Let g_0 be the genus of S_0. At $p\in S$ and at $p_0=\pi(p)\in S_0$, by a suitable choice of local parameters, π is represented locally by $z_0=z^\nu$, where ν is a positive integer, z and z_0 are the local parameters at p, p_0, respectively. If $\nu>1$, p is called a branch point of multiplicity ν. If $\pi(p_1)=\pi(p_2)$ ($p_1, p_2\in S$), then multiplicity at p_1 is equal to that at p_2. Therefore we can define the multiplicity over $p_0\in S_0$ by the multiplicity at $p\in\pi^{-1}(p_0)$. Let t be the number of the points in S_0 which are the projections of all branch points. We call the set of integers g_0 and all multiplicities $\nu_1, \cdots, \nu_t$ the signature of G and denote it by $(g_0; \nu_1, \cdots, \nu_t)$. Without loss of generality, we may assume $\nu_1\leqq\nu_2\leqq\cdots\leqq\nu_t$. For simplicity, we shall denote $(0; \nu_1, \cdots, \nu_t)$ by $(\nu_1, \cdots, \nu_t)$.

§3. **Lemmas.**

Lemma 1 (Wiman [9], Nakagawa [5]). *If ν is a multiplicity of G then $2\leqq\nu\leqq 4g+2$.*

Lemma 2. *There exists neither an automorphism of order* 7 *nor of order* 9 *on any compact Riemann surface of genus* 5.

Lemma 3. *For all $k\geqq 0$, $N(5, k)\geqq 8$.*

We are going to determine whether the automorphism group with a given signature exists or not on a compact Riemann surface of genus 5. By Lemma 3, it is not necessary to consider the groups of order $N\leqq 8$. We assume $N>8$. By the Riemann-Hurwitz relation, an easy calculation