45. Representations of Hecke Algebras on Virtual Character Modules of a Semisimple Lie Group*)

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(Communicated by Shokichi IYANAGA, M. J. A., April 14, 1986)

§1. **Introduction.** The purpose of this note is to present, together with some further consequences, the main results of our Thesis [6] to which we shall leave detailed descriptions. Let G be a connected semisimple Lie group with finite centre, which is assumed to be "acceptable", i.e., satisfying certain natural conditions for technical reasons. Let $\mathfrak{g}$ be the Lie algebra of G and $\mathfrak{g}_C$ its complexification. We denote by $U(\mathfrak{g}_C)$ the enveloping algebra of $\mathfrak{g}_C$ and by $\mathfrak{Z}$ the centre of $U(\mathfrak{g}_C)$.

Let Car(G) be the set of all the conjugacy classes of Cartan subgroups of G. Then $\sharp$Car(G) is finite. Take $[H]\in$ Car(G), where $[H]$ means the conjugacy class of a Cartan subgroup H. We fix H as a representative of the class. Let $\mathfrak{h}$ and $\mathfrak{h}_C$ be the Lie algebra of H and its complexification respectively. Put $W=W(\mathfrak{g}_C, \mathfrak{h}_C)$, the complex Weyl group. For each $\lambda\in\mathfrak{h}_C^*$, we define a subgroup $W_H(\lambda)$ and a subset $W_H^{\sim}(\lambda)$ of W as in [4, pp. 724–725]. We call $W_H(\lambda)$ the *integral Weyl group* for H and λ. We also define the fixed subgroup of λ as $W_\lambda=\{w\in W \mid w\lambda=\lambda\}$.

For each irreducible admissible representation $(\pi, \mathfrak{H})$ of G on a Hilbert space $\mathfrak{H}$, there corresponds to $\chi\in \mathrm{Hom}_{alg}(\mathfrak{Z}, C)$ so-called infinitesimal character of π. By Harish-Chandra homomorphism, $\mathfrak{Z}$ is isomorphic to $U(\mathfrak{h}_C)^W$ as an algebra, where $U(\mathfrak{h}_C)^W$ denotes the set of all the W-fixed elements in $U(\mathfrak{h}_C)$. Since $\mathrm{Hom}_{alg}(U(\mathfrak{h}_C)^W, C)\simeq\mathfrak{h}_C^*/W$, there exists $\lambda\in\mathfrak{h}_C^*$ which naturally defines χ. We denote this as $\chi=\chi_\lambda$. Remark that $w\lambda(w\in W)$ also defines χ. Let Mod(χ) be the set of irreducible admissible representations of G with infinitesimal character χ. For $\pi\in$ Mod(χ), one can define the character θ_π which is a constant coefficient invariant eigendistribution on G [5]. Put

$$V(\chi)=\langle\theta_\pi \mid \pi\in \mathrm{Mod}(\chi)\rangle \qquad \text{(generated as a vector space over } C).$$

Then $V(\chi)$ is finite dimensional. We can define subspaces $V_H(\chi)$ ($[H]\in$ Car(G)), denoted by $V_H(\lambda)$ in [4], and we have the direct sum decomposition [4, p. 726]

$$V(\chi)=\bigoplus_{[H]\in \mathrm{Car}(G)} V_H(\chi).$$

§2. **Representations of Hecke algebras $\mathcal{H}(W_H(\lambda), W_\lambda)$ on $V_H(\chi)$.** Fix a Cartan subgroup H. In the preceding paper [4, §3], we define a repre-

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*) This research was partially supported by Grant-in-Aid for Scientific Research (No. 60790018), Ministry of Education, Science and Culture of Japan.