41. *Homology of a Local System on the Complement of Hyperplanes*

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1. Introduction and statement of results. Let $\{A_j\}_{1\leqslant j\leqslant m}$ be a finite family of complex affine hyperplanes in $\boldsymbol{C}^n$ and let $\mathcal{L}$ be a local system on the complement $X=\boldsymbol{C}^n-\bigcup_{j=1}^m A_j$. The vanishing of homology $H_j(X, \mathcal{L})$, $j\neq n$, for a "generic" local system $\mathcal{L}$ was treated by K. Aomoto [2] and M. Kita-M. Noumi [7] from different points of view. The object of this note is to give a simple criterion for such vanishing of homology and to give a basis of $H_n(X, \mathcal{L})$. We denote by f_j, $1\leqslant j\leqslant m$, a linear form with $\operatorname{Ker} f_j=A_j$ and let A_{m+1} denote the hyperplane at infinity. We consider a regular connection of the form $\Omega=\sum_{j=1}^m P_j d\log f_j$, $P_j\in \operatorname{End}(V)$, where V is a finite dimensional complex vector space. Let us observe that the connection Ω is integrable if and only if $[P_{j_\nu}, P_{j_1}+\cdots+P_{j_q}]=0$, $1\leqslant\nu\leqslant q$, for any maximal family $\{A_{j_\nu}\}_{1\leqslant\nu\leqslant q}$ such that $\operatorname{codim}_{\boldsymbol{C}}[A_{j_1}\cap\cdots\cap A_{j_q}]=2$ (see [1]). These relations are related to the lower central series of the fundamental group of X (see [8], [9]). Let P_{m+1} denote the residue along A_{m+1}. We have $P_1+\cdots+P_{m+1}=0$.

Let us suppose that Ω is integrable in the followings. The connection Ω is said to be *generic with respect to the hyperplanes* $\{A_j\}_{1\leqslant j\leqslant m+1}$ if the following conditions are satisfied:

(1.1) (i) Any eigenvalue of $P_j, 1\leqslant j\leqslant m+1$, is not an integer.
(ii) For any maximal subfamily $\{A_{j_\nu}\}_{1\leqslant\nu\leqslant q}$, such that $\operatorname{codim}_{\boldsymbol{C}}[A_{j_1}\cap\cdots\cap A_{j_q}]=r$ with some $r<q$, any eigenvalue of $P_{j_1}+\cdots+P_{j_q}$ is not an integer.

The solutions of the system of differential equations $dY+\Omega . Y=0$ defines a local system $\mathcal{L}$ on X, which determines a homomorphism $\rho:\pi_1(X, x_0)\to \operatorname{Aut}(\mathcal{L}_{x_0})$. Let $\tilde{X}$ be the universal covering of X. The homology $H_j(X, \mathcal{L})$ is defined to be the j-th homology of the complex $C.(\tilde{X})\otimes_{\boldsymbol{Z}[G]}\mathcal{L}_{x_0}$, where $G=\pi_1(X, x_0)$ and the space of chains of $\tilde{X}$ is considered as a right $\boldsymbol{Z}[G]$-module via covering transformations and $\mathcal{L}_{x_0}$ is a left $\boldsymbol{Z}[G]$-module via ρ. The homology of the locally finite (possibly infinite) chains is defined in the same way and we denote it by $H_j^{lf}(X, \mathcal{L})$.

Theorem 1. *Let us suppose that the integrable connection* $\Omega=\sum_{j=1}^m P_j \times d\log f_j$ *is generic with respect to the hyperplanes* $\{A_j\}_{1\leqslant j\leqslant m+1}$ *in the sense of* (1.1). *Let* $\mathcal{L}$ *denote the local system over* $X=\boldsymbol{C}^n-\bigcup_{j=1}^m A_j$ *associated with* Ω. *Then we have an isomorphism*

$$H_j^{lf}(X, \mathcal{L})\cong H_j(X, \mathcal{L}) \qquad \textit{for any } j, \tag{1.2}$$