40. *Multi-Tensors of Differential Forms on the Siegel Modular Variety and on its Subvarieties*

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Introduction. Let $A_n=H_n/\Gamma_n$, where H_n is the Siegel space $\{Z\in M_n(\boldsymbol{C})\,|\,{}^tZ=Z,\ \mathrm{Im}\,Z>0\}$, and $\Gamma_n=Sp_{2n}(\boldsymbol{Z})$. A_n is shown to be of general type for $n\geqq 9$ by Tai [5] ($n=8$ by Freitag [2], $n=7$ by Mumford [4]). Subvarieties of A_n are expected to have the same property if they are not too special. We have the following theorem. The details of the proof are included in Tsuyumine [9].

Theorem. *Let $n\geqq 10$. Then any subvariety in A_n of codimension one is of general type.*

We have the following corollary to this theorem (cf. Freitag [3]). We denote by $\Gamma_n(l)$ the principal congruence subgroup of level l, and by $A_{n,l}$ the quotient space $H_n/\Gamma_n(l)$.

Corollary. *Let $n\geqq 10$. Then the birational automorphism group of $A_{n,l}$ equals* $\mathrm{Aut}(A_{n,l})\simeq\Gamma_n/\pm\Gamma_n(l)$. *In particular, A_n has no non-trivial birational automorphism.*

§1. **Preliminaries.** The symplectic group $Sp_{2n}(\boldsymbol{R})$ acts on H_n by the usual symplectic substitution:

$$Z\longrightarrow MZ=(AZ+B)(CZ+D)^{-1},$$
$$M=\begin{pmatrix}A & B\\ C & D\end{pmatrix}\in Sp_{2n}(\boldsymbol{R}).$$

Let $Z=(z_{ij})$, and let

$$\omega_{ij}=(-1)^{i+j}e_{ij}dz_{11}\wedge dz_{12}\wedge\cdots\wedge \check{dz_{ij}}\wedge\cdots\wedge dz_{nn},\qquad e_{ij}=\begin{cases}1 & i\neq j,\\ 2 & i=j,\end{cases}$$

for $1\leqq i\leqq j\leqq n$. Let $\omega=(\omega_{ij})$. Then we have

$$M\cdot\omega=|CZ+D|^{-n-1}(CZ+D)\omega^t(CZ+D),$$

and so

$$M\cdot\omega^{\otimes r}=|CZ+D|^{-r(n+1)}(CZ+D)^{\otimes r}\omega^{\otimes r\,t}(CZ+D)^{\otimes r}.$$

A Siegel modular form f admits the Fourier expansion $f(Z)=\sum_{S\geqq 0}a(S)\boldsymbol{e}(\mathrm{tr}\,((1/2)SZ))$, $\boldsymbol{e}(\ \ )$ standing for $\exp(2\pi\sqrt{-1}\ \ )$. f is said to vanish to order α (at the cusp) if α is the minimum integer such that $a(S)=0$ for S with $\min_{g\in Z^n,\neq 0}\{(1/2)S[g]\}<\alpha$, $S[g]$ denoting ${}^tgSg$. We denote it by ord(f).

§2. **Theta series.** Let m be an integer with $m\geqq 2(n-1)$, and let η be a complex $m\times(n-1)$ matrix satisfying both ${}^t\eta\eta=0$ and rank $\eta=n-1$. η_i $(1\leqq i\leqq n)$ denotes an $(n-1)\times n$ matrix given by