# 4. *Fock Space Representations of Virasoro Algebra and Intertwining Operators*

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§ 0. In this note, we construct the Fock space representations of the Virasoro algebra $\mathcal{L}$ and intertwining operators between them in the explicit form, and give the analogous determinant formula for them as for the Verma modules (see V. G. Kac [4]). Proofs and details will be given in the forthcoming paper [6].

§ 1. The Virasoro algebra $\mathcal{L}$ is the Lie algebra over the complex number field $\boldsymbol{C}$ of the following form:

$$\mathcal{L}=\sum_{n\in \boldsymbol{Z}} \boldsymbol{C}e_n \oplus \boldsymbol{C}e_0',$$

with the relations: for any $m, n \in \boldsymbol{Z}$

$$\begin{cases}[e_n, e_m]=(m-n)e_{m+n}+((m^3-m)/12)\delta_{m+n,0}e_0', \\ [e_0', e_m]=0.\end{cases}$$

This is a unique central extension of the Lie algebra $\mathcal{L}'$ of trigonometric polynomial vector fields on the circle:

$$\mathcal{L}'=\sum_{n\in \boldsymbol{Z}} \boldsymbol{C}l_n\,;\ [l_n, l_m]=(m-n)l_{m+n}\ (m, n\in \boldsymbol{Z}) \qquad (l_n=z^{n+1}(d/dz)).$$

Let $\mathfrak{h}=\boldsymbol{C}e_0\oplus \boldsymbol{C}e_0'$ be the abelian subalgebra of $\mathcal{L}$ of maximal dimension. For each $(h, c)\in \boldsymbol{C}^2\cong\mathfrak{h}^*$ the dual of $\mathfrak{h}$, we can define the Verma module $M(h, c)$ and its dual $M^\dagger(h, c)$ as follows. $M(h, c)$ and $M^\dagger(h, c)$ are the left and right $\mathcal{L}$-modules with cyclic vectors $|h, c\rangle$ and $\langle c, h|$ with following fundamental relations respectively:

$$e_{-n}|h, c\rangle=0\ (n\geq 1);\quad e_0|h, c\rangle=h|h, c\rangle,\quad e_0'|h, c\rangle=c|h, c\rangle;$$
$$\langle c, h|e_n=0\quad (n\geq 1);\quad \langle c, h|e_0=\langle c, h|h,\quad \langle c, h|e_0'=\langle c, h|c.$$

V. G. Kac [4] studied these $\mathcal{L}$-modules and obtained the formula concerning the determinants of the matrices of their vacuum expectation values. By this Kac's determinant formula, F. L. Feigin and D. B. Fuks [3] determined the composition series of $M(h, c)$.

§ 2. Consider the associative algebra $\mathcal{A}$ over $\boldsymbol{C}$ generated by $\{p_n (n\in \boldsymbol{Z}), \Lambda\}$ with the following Bose commutation relations:

$$[p_m, p_n]=n\delta_{m+n,0}id\,;\quad [\Lambda, p_m]=0\quad (m, n\in \boldsymbol{Z}).$$

And consider the following operators in a completion $\hat{\mathcal{A}}$ of $\mathcal{A}$:

$$L_n=(p_0-n\Lambda)p_n+\frac{1}{2}\sum_{j=1}^{n-1} p_j p_{n-j}+\sum_{j\geq 1} p_{n+j}p_{-j} \qquad (n\geq 1);$$

$$L_{-n}=(p_0+n\Lambda)p_{-n}+\frac{1}{2}\sum_{j=1}^{n-1} p_{-j}p_{j-n}+\sum_{j\geq 1} p_j p_{-n-j} \qquad (n\geq 1);$$

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