# 34. On a Local Existence Theorem for Quasilinear Hyperbolic Mixed Problems with Neumann Type Boundary Conditions

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(Communicated by Kôsaku YOSIDA, M. J. A., April 14, 1986)

§1. **Introduction.** Let $S$ be a $C^\infty$ and compact hypersurface in $\boldsymbol{R}^n$ and $\Omega$ be the interior or exterior domain of $S$. We shall consider the local existence in time of classical solutions for the following Neumann problem:

$$\text{(N.P)}\quad\begin{cases} \underset{\sim}{P}(\underset{\sim}{u})_a=\partial_t^2 u_a-\sum_{i=1}^n \partial_i(A_{ia}(t,x,\bar{D}_x^1\underset{\sim}{u}))+\Phi_a(t,x,\bar{D}^1\underset{\sim}{u}) \\ \qquad =f_a(t,x) \quad \text{in } [0,T]\times\Omega, \\ \underset{\sim}{Q}(\underset{\sim}{u})_a=\sum_{i=1}^n \nu_i(x)A_{ia}(t,x,\bar{D}_x^1\underset{\sim}{u})+\Psi_a(t,x,\underset{\sim}{u})=g_a(t,x) \quad \text{on } [0,T]\times S, \\ u_a(0,x)=u_a^0(x), \qquad (\partial_t u_a)(0,x)=u_a^1(x) \quad \text{in } \Omega \end{cases}$$

for $a=1,\cdots,m$. Here, $\underset{\sim}{u}=(u_1,\cdots,u_m)$, $\bar{D}_x^1\underset{\sim}{u}=(\partial_i u_a\,;\, i=1,\cdots,n,\ a=1,\cdots,m,\ u_i\,;\, i=1,\cdots,m)$, $\bar{D}^1\underset{\sim}{u}=(\partial_t u_a\,;\, a=1,\cdots,m,\ \bar{D}_x^1\underset{\sim}{u})$, $\partial_t=\partial/\partial t$, $\partial_j=\partial/\partial x_j$ and $\nu(x)=(\nu_1(x),\cdots,\nu_n(x))$ is the outer unit normal of $S$ at $x\in S$. We also try to obtain a sharp energy estimate of the regularity of solution in terms of the data and the operators $\underset{\sim}{P}$, $\underset{\sim}{Q}$.

Our result has applications to the classical nonlinear wave equation with Neumann or third kind boundary condition and the equation of motion describing the small deformation of a homogeneous, isotropic, hyperelastic material under action of gravity and surface force of dead load type.

Although there are many works for the Cauchy problem and the Dirichlet problem (see [1]~[6]), it seems that the lack of the good estimate for the linearized Neumann problem such as for the Cauchy problem or the Dirichlet problem has kept away from proving the local existence theorem for the Neumann problem. The deficiency of the estimate is the derivative loss which breaks down the usual iteration process. Here, in order to avoid any misunderstanding, we add a comment. Namely, if a rough estimate for the regularity of the solution is enough and if we restrict to the case $m=1$, a global existence theorem is proved in [7] by using the Nash-Moser technique.

The idea of the proof is to introduce the new unknown $\partial_t\underset{\sim}{u}$ and replace (N.P) by the Neumann problem for some equivalent hyperbolic-elliptic system with respect to the unknowns $(\underset{\sim}{u},\partial_t\underset{\sim}{u})$. Then we can get an estimate which is good enough to carry out the usual iteration process for the new Neumann problem.

§2. **Result and examples.** Before stating our main result, we list

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