32. *Relative Zariski Decomposition on Higher Dimensional Algebraic Varieties*

By Atsushi MORIWAKI
Department of Mathematics, Faculty of Science, Kyoto University

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0. Introduction. The purpose of this note is to state several results in my Master Thesis [7]. The details will be published elsewhere. The main theorem of this note is Theorem 3. By this theorem, if K_X has a good Zariski decomposition, then the canonical ring of X is finitely generated. Theorem 1 and Theorem 2 are key theorems to prove Theorem 3. Theorem 5 is a characterization of a nef and good divisor by μ_x. All varieties in this note are assumed to be defined over an algebraically closed field of characteristic zero.

1. Notation. Let X be an algebraic scheme. We denote the group of Cartier divisors on X by $\mathrm{Div}(X)$. For a non-zero rational function ϕ on X, the principal Cartier divisor defined by ϕ is denoted by $\mathrm{div}(\phi)$. For $D_1, D_2 \in \mathrm{Div}(X)\otimes \boldsymbol{R}$, we say D_1 is *$\boldsymbol{R}$-linear equivalent* to D_2, which is denoted by $D_1 \sim_{\boldsymbol{R}} D_2$, if there exists a positive integer m and exists a non-zero rational function ϕ on X such that $D_1 = D_2 + (1/m)\,\mathrm{div}(\phi)$. For a real number a, the lounding-up, the lounding-down, the nearest integer and the fractional part of a are denoted by $\ulcorner a \urcorner$, $[a]$, $\langle a \rangle$ and $\{a\}$ respectively, where in case $\{a\}=1/2$, we define $\langle a \rangle = \ulcorner a \urcorner$ if $a>0$, $\langle a \rangle = [a]$ if $a<0$. From now on, we assume X is non-singular. Let D be an element of $\mathrm{Div}(X)\otimes \boldsymbol{R}$ and $D=\sum_i a_i D_i$ the irreducible decomposition of D. Then we set $\ulcorner D \urcorner = \sum_i \ulcorner a_i \urcorner D_i$, $[D]=\sum_i [a_i] D_i$, $\langle D \rangle = \sum_i \langle a_i \rangle D_i$ and $\{D\} = \sum_i \{a_i\} D_i$. Let $\mathcal{J}$ be an ideal sheaf of $\mathcal{O}_X$ and x a point of X (not necessarily closed). Then we define

$$\mathrm{ord}_x(\mathcal{J}) = \max\{a \in \boldsymbol{N}\cup\{\infty\} \mid \mathcal{J}\mathcal{O}_{X,x} \subseteqq m_x^a\} \text{ and } \mathrm{ord}_x(D) = \textstyle\sum_i a_i\, \mathrm{ord}_x(\mathcal{O}_X(-D_i)),$$

where m_x is the maximal ideal of $\mathcal{O}_{X,x}$. We furthermore assume X is complete. We set $\kappa(X, D) = \max_m \{\kappa(X, [mD])\}$. If $\kappa(X, D) = \dim X$, D is called *big*. D is called *good* if there exists a birational morphism $\pi: Y \to X$ of non-singular complete varieties and exists a fiber space $h: Y \to Z$ of non-singular complete varieties such that $\pi^*(D) \sim_{\boldsymbol{R}} h^*(M)$ for some big element M of $\mathrm{Div}(Z)\otimes \boldsymbol{R}$. Next, we consider the relative case. Let X be a non-singular algebraic variety, S an algebraic variety, $f: X \to S$ a proper surjective morphism. For $D \in \mathrm{Div}(X)\otimes \boldsymbol{R}$, we set

$$E(X/S, D) = \{n \in \boldsymbol{N}\backslash\{0\} \mid f_*\mathcal{O}_X([nD]) \neq 0\}.$$

D is called *f-nef* if $(D\cdot C) \geqq 0$ for any complete curve C on X such that $f(C)$ is a point. D is called *f-big* (resp. *f-good*) if $D|_{X_\eta}$ is a big (resp. good) element of $\mathrm{Div}(X_\eta)\otimes \boldsymbol{R}$, where X_η is the generic fiber of f. For a Cartier