1. *Hodge Structure and Holonomic Systems*

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§0. **Introduction.** **0.1.** Let X be a compact complex variety of type C in the sense of Fujiki [3], and X_0 a non-singular Zariski open subset of X. Let (H_Z, F) be a variation of Hodge structure on X_0 of weight w. The well-known conjecture (e.g. [1]) is that the cohomology group $H^n(X; {}^\pi H_Q)$ has a Hodge structure of weight $n+w$.

0.2. Toward this direction, we gave the affirmative answer in [4] when X is a non-singular Kähler manifold and $S=X\setminus X_0$ is a normally crossing hypersurface. It was also given by Cattani, Kaplan and Schmid [2] independently. But, the Hodge filtration of the cohomclogy groups was not given in an algebro-geometric way.

0.3. In this article we announce how to construct the Hodge filtration in an algebraic way.

§1. The starting point of the construction is the following method.

1.1. **Sandwich method.** Let X be a topological space, and $(K^\cdot, F(K^\cdot))$ a filtered complex of sheaves on X. Suppose

(1.1) $\qquad H^n(X; F^p(K^\cdot)) \longrightarrow H^n(X; K^\cdot)$ is injective.

Let $(K_1^\cdot, F(K_1^\cdot))$, $(K_2^\cdot, F(K_2^\cdot))$ be two filtered complexes and assume that we have filtered morphisms $K_1^\cdot \xrightarrow{f} K^\cdot \xrightarrow{g} K_2^\cdot$. Now we make the following hypothesis.

(1.2) $K_1^\cdot \xrightarrow{f} K^\cdot \xrightarrow{g} K_2^\cdot$ are quasi-isomorphisms forgetting filtrations.

(1.3) $K_1^\cdot \longrightarrow K_2^\cdot$ is a filtered quasi-isomorphism.

Then

$$H^n(X; F^p(K_1^\cdot)) \xrightarrow{\sim} H^n(X; F^p(K^\cdot)) \xrightarrow{\sim} H^n(X; F^p(K_2^\cdot)),$$

and therefore $H^n(X; F^p(K_1^\cdot))$ gives the same filtration on $H^n(X; K^\cdot)$ as the one coming from $F(K^\cdot)$.

1.2. Let us return to the situation of §0.2. Then we know that the complex $\mathcal{L}^\cdot(H)_{(2)}$ of square-integrable sections is filtered and that it satisfies (1.1). Further the image of the homomorphism gives the Hodge filtration of the intermediate cohomology groups. In order to illustrate our method, we shall show that the sandwich method combined with these facts gives another proof of a result of Zucker [7] in the one-variable case. We employ the same notations as in §4 of [7]. We shall construct $K_1^\cdot$ and $K_2^\cdot$ as follows:

(1.4) $$K^\cdot = \mathcal{L}^\cdot(H)_{(2)},$$