3. *On Topological Dynamical Systems with Discrete Spectrum*

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§1. **Main results.** Throughout this note (X, T) is a topological dynamical system, i.e., a pair of a compact Hausdorff space X and a continuous map T of X to itself. Let $C(X)$ be the Banach space of all continuous complex functions on X with the usual supremum norm. By $E(X, T)$ [resp. $\sigma(X, T)$] we denote the set of all eigenfunctions [resp. eigenvalues] of U_T defined by $U_T(f)=f\circ T(f\in C(X))$. We say that (X, T) has *discrete spectrum* if the norm closed linear span of $E(X, T)$ is identical with $C(X)$. For any fixed $x\in X$ we put $O_T(x)=\{T^n x\,;\ n\in \boldsymbol{N}\}$ and $O_T^+(x)=\{T^n x\,;\ n\in \boldsymbol{Z}^+\}$, where $\boldsymbol{N}[\boldsymbol{Z}^+]$ is the set of all nonnegative [positive] integers. (X, T) is said to be *topologically transitive* if there exists some $p\in X$ for which $O_T(p)$ is dense in X. We distinguish the topological transitivity for (X, T) into the following two cases:

(A) There exists some $p\in X$ for which $O_T^+(p)$ is dense in X.

(B) There exists some $p\in X$ for which $O_T(p)$ is dense in X, and $O_T^+(x)$ is not dense in X for all $x\in X$.

The purpose of this paper is to clarify the structure of topologically transitive (X, T) with discrete spectrum. We say that (X, T) is *topologically conjugate* to a topological dynamical system (Y, S), in symbol $(X, T)\cong(Y, S)$, if there exists a homeomorphism ϕ of X onto Y such that $\phi\circ T=S\circ\phi$. Let $(X, T)\cong(Y, S)$. Then $\sigma(X, T)=\sigma(Y, S)$, (X, T) has discrete spectrum if and only if so has (Y, S), and further (X, T) satisfies (A) [(B)] if and only if (Y, S) satisfies (A) [(B)].

Let G be a compact abelian semigroup, $a\in G$ and L_a the translation on G defined by a. Then we get a topological dynamical system (G, L_a). Let $G_e=G\cup\{e\}$ be the adjunction of an identity e to G. This is also a compact abelian semigroup in which e is an isolated point. A semicharacter of G is a continuous function χ on G such that $\chi(g)\neq 0$ for some $g\in G$ and $\chi(st)=\chi(s)\chi(t)$ for all s, t in G. By $\hat{G}$ we denote the set of all semicharacters of G. G is said to be *separative* if for any distinct $s, t\in G$ there exists $\chi\in\hat{G}$ with $\chi(s)\neq\chi(t)$. As seen easily G_e is separative if and only if so is G. Further if G is separative, then the norm closed linear span of $\hat{G}$ is identical with $C(G)$. If there exists some $a\in G$ such that $\{a^n\,;\ n\in\boldsymbol{Z}^+\}$ is dense in G, then G is called a monotetic semigroup with the generator a. Under the above notations and terminology our main results are stated as follows.

Theorem 1. *(X, T) has discrete spectrum and satisfies the condition*