21. *On Tight t-designs in Compact Symmetric Spaces of Rank One*

By Eiichi BANNAI†),*) and Stuart G. HOGGAR**)

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We announce the following result. For the definition of tight t-designs, see § 1.

Theorem 1. *There exists an absolute constant t_0 which satisfies the following: if X is a tight t-design in one of the complex projective spaces $P^d(C)$ $(d=4, 6, 8, \cdots)$ or the quaternion projective spaces $P^d(H)$ $(d=8, 12, 16, \cdots)$ then we have $t \leq t_0$.*

Since the corresponding results for the other compact rank 1 symmetric spaces are already obtained (see [1], [2], [5]), we have the following.

Corollary to Theorem 1. *There exists another absolute constant t_0 which satisfies the following: if X is a tight t-design in one of the connected compact rank 1 symmetric spaces of (topological) dimension $d \geq 2$, then we have $t \leq t_0$. (Here we need t_0 to be at least 11 as there exists a tight 11-design in S^{23}.)*

We expect that the actual value of t_0 in Theorem 1 can be very small (i.e., something like 5 although it may not be exactly 5). The determination of the exact value of t_0, which is very involved, will be treated in a subsequent full paper which is now being prepared by us.

§ 1. Preliminaries. Let S be a connected compact symmetric space of rank 1. That is, S is one of the following spaces: sphere S^d, projective spaces $P^d(K)$ where K is one of the real field $\boldsymbol{R}$ $(d=2, 3, 4, \cdots)$, complex field $\boldsymbol{C}$ $(d=4, 6, 8, \cdots)$, quaternion field $\boldsymbol{H}$ $(d=8, 12, 16, \cdots)$ or the Cayley octanions $\boldsymbol{O}$ $(d=16)$. Then $S=H\backslash G$ for a suitable pair of a compact Lie group G and its closed subgroup H. The space $L^2(S)$ is decomposed into the direct sum of irreducible G-spaces V_i (i.e. $L^2(S)=V_0 \oplus V_1 \oplus V_2 \oplus \cdots$) where V_i gives the i-th "spherical" representation of G. The dimension of V_i is finite and is denoted by m_i (cf. § 2).

A finite non-empty subset X of S is called a *t-design* in S if $\sum_{x \in X} f(x) = 0$ for any function $f \in V_1 \oplus V_2 \oplus \cdots \oplus V_t$. Note that for each t and each S, the existence of *t-designs* X in S is guaranteed by Seymour-Zaslavsky [17]. The reader is referred to [5], [6], [14], [16], etc. for the examples and the fundamental properties of t-designs in S.

Let $d(x, y)$ be the distance function on S, and let δ be the diameter of S, i.e., $\delta = \mathrm{Max}_{x,y \in X}\, d(x, y)$. Let $A(X) := \{d(x, y) \mid x, y \in X, x \neq y\}$. If $|A(X)|$

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*) The Ohio State University; Queen Mary College, University of London.
**) University of Glasgow.