20. On Certain Elliptic Conjugacy Classes of the Siegel Modular Group

By Ki-ichiro HASHIMOTO
Department of Mathematics, Waseda University

(Communicated by Shokichi IYANAGA, M. J. A., March 12, 1985)

0. In this note we describe some results on the parametrization of the elliptic conjugacy classes in $\Gamma_n=\mathrm{Sp}(n, \boldsymbol{Z})$, the Siegel modular group of degree n, of the elements whose minimal polynomials are irreducible over $\boldsymbol{Q}$, hence are cyclotomic polynomials $\Phi_m(X)$. Our first result (Theorem 1) shows the bijective correspondence of the conjugacy classes in Γ_n with $\Phi_m(X)$ and the isometric classes of (skew-)hermitian forms over the ring of integers of the splitting field K of $\Phi_m(X)$, which generalizes our previous result [4]. Then we study in more details the case of $\varphi(m)=2n$, where the elements are regular. Especially, we show that the number of such conjugacy classes in Γ_n is equal to $h^-(K)$, the relative class number of K, multiplied by a power of 2 which is the number of "*integral*" classes in $\mathrm{Sp}(n, \boldsymbol{Q})$ or $\mathrm{Sp}(n, \boldsymbol{R})$. This refines a result of Midorikawa [6]. In Theorem 3, we characterize the integral conjugacy classes in $\mathrm{Sp}(n, \boldsymbol{R})$ in terms of their eigenvalues as an element of $\mathrm{U}(n)$, the maximal compact subgroup. There are two proofs for our results, one of which is an application of our previous result [3]. Details will appear elsewhere.

1. **Notations.**

$\sharp(S):=$ the cardinality of a finite set S.

$\Phi_m(X):=\prod_{d\mid m}(X^d-1)^{\mu(m/d)}$, m-th cyclotomic polynomial.

$K:=\boldsymbol{Q}(\zeta_m)$, $\zeta_m=e^{2\pi i/m}$; $K_o:=\boldsymbol{Q}(\zeta_m+\zeta_m^{-1})$.

$O_K, O_{K_o}:=$ the ring of integers of K, K_o.

$\mathfrak{d}=\mathfrak{d}(K/\boldsymbol{Q})$, the Differente of $K/\boldsymbol{Q}$.

$t:=\sharp\{\mathfrak{p}$; prime ideals in K_o, ramified in $K/K_o\}$.

$=1$, or 0 according as m is a prime power or not ($m\not\equiv 2$, mod 4).

For a commutative ring A with 1,

$$\mathrm{Sp}(n, A):=\left\{g\in \mathrm{SL}_{2n}(A)\,;\, gJ_n{}^tg=J_n,\ J_n=\begin{pmatrix}0 & -1_n\\ 1_n & 0\end{pmatrix}\right\}.$$

$\Gamma_n:=\mathrm{Sp}(n, \boldsymbol{Z})$, the Siegel modular group of degree n.

$G_{\boldsymbol{Q}}:=\mathrm{Sp}(n, \boldsymbol{Q})$, $G_{\boldsymbol{R}}:=\mathrm{Sp}(n, \boldsymbol{R})$.

For a subgroup H of $G_{\boldsymbol{R}}$,

$H(\Phi_m):=\{g\in H$; g is semi-simple with minimal polynomial $\Phi_m(X)\}$.

$C_H(g):=\{x^{-1}gx\,;\, x\in H\}$, the H-conjugacy class of g.

$H(\Phi_m)/\!\!/H:=$ the set of H-conjugacy classes in $H(\Phi_m)$.

2. **Results.** We assume, throughout this note, that m ($\not\equiv 2$, mod 4) is a positive integer satisfying $2n\equiv 0 \bmod \varphi(m)$, where $\varphi(m)=\sharp(\boldsymbol{Z}/m\boldsymbol{Z})^*$.