

15. Diffeomorphism Types of Elliptic Surfaces

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§ 1. Statement of results. The purpose of this note is to announce some results concerning diffeomorphism types of Kodaira's elliptic surfaces [2]. Elliptic surfaces we consider here will satisfy the following conditions :

- 1) No fiber contains an exceptional curve of the first kind ;
- 2) at least one singular fiber exists ;
- 3) there are no multiple singular fibers.

Theorem 1. *Let $\Phi_i: M_i \rightarrow B_i$ ($i=1, 2$) be elliptic surfaces satisfying the conditions 1), 2), 3). Then there exists an orientation preserving diffeomorphism $f: M_1 \rightarrow M_2$ if and only if $g(B_1)=g(B_2)$ and $e(M_1)=e(M_2)$, where $g(B)$ and $e(M)$ denote the genus of the base curve B and the Euler number of the total space M , respectively.*

This result extends Kas' theorem [1] which deals with the case $g(B_i)=0$. See also Moishezon [7].

If an elliptic surface $\Phi: M \rightarrow B$ satisfies the conditions 1), 2), 3), then by deforming the projection map Φ if necessary, we may (and will) assume that all the singular fibers are of type I_1 ([1], [7]). Let x_1, x_2, \dots, x_n be the singular loci. We choose a base point $x_0 \in B - \{x_1, \dots, x_n\}$ and a basis (e_1, e_2) of $H_1(\Phi^{-1}(x_0); \mathbf{Z})$. Then the monodromy representation

$$\rho: \pi_1(B - \{x_1, \dots, x_n\}, x_0) \longrightarrow SL(2, \mathbf{Z})$$

is well-defined.

We draw loops $L_1, M_1, \dots, L_n, M_n, g_1, g_2, \dots, g_n$ on $B - \{x_1, \dots, x_n\}$ as shown in Fig. 1 (in case $g=g(B)=2$), g_i being a loop based at x_0 which goes round x_i once.

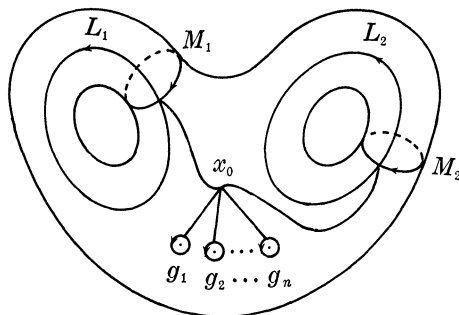


Fig. 1

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