2. Instability of Periodic Solutions of Some Evolution Equations Governed by Time-Dependent Subdifferential Operators

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Let H be a Hilbert space with norm $|\cdot|$, and $\Phi(H)$ be the set of all proper lower semicontinuous convex functions from H into $(-\infty, \infty]$. Given a T-periodic mapping $t \mapsto \phi^t$ from $\boldsymbol{R}$ into $\Phi(H)$, and a T-periodic function f in $L^2_{\mathrm{loc}}(\boldsymbol{R}; H)$ (i.e. $\phi^{t+T} = \phi^t$ for $t \in \boldsymbol{R}$, and $f(t+T) = f(t)$ for a.e. $t \in \boldsymbol{R}$), we consider the equation

(E) $$u'(t) + \partial\phi^t(u(t)) \ni f(t), \qquad t \in J,$$

where J is an interval in $\boldsymbol{R}$, $u'(t) = (d/dt)u(t)$ and $\partial\phi^t$ is the subdifferential of ϕ^t. For related studies on (E) we refer to [2, 4, 7, 8, 11, 12, 13].

In [3], Baillon and Haraux treated the time-independent case of ϕ^t, i.e. $\phi^t \equiv \phi$, and proved that any solution on $J = [t_0, \infty)$ is asymptotically T-periodic in the weak topology of H and the difference of any two T-periodic solutions is a constant vector on $\boldsymbol{R}$. Subsequently, Haraux [5] and Ishii [6] discussed the equation from the same viewpoint as in [3], when $\phi^t \equiv \phi$ and f is almost periodic on $\boldsymbol{R}$. In this paper we shall show by a simple example in 3-dimensional space that the equation with the time-dependent ϕ^t is essentially different in nature from that with the time-independent $\phi^t \equiv \phi$.

1. A flow in 3-dimensional space. We take 3-dimensional space $\boldsymbol{R}^3$ as H, and denote by $x = (x_1, x_2, x_3)$ a generic point in $\boldsymbol{R}^3$. Now, for each $t \in \boldsymbol{R}$ and $\theta \in [0, \infty)$, let us consider the operator $R_\theta(t)$ from the x_1x_2-plane $X_0 = \{(x_1, x_2, 0); x_1, x_2 \in \boldsymbol{R}\}$ into $\boldsymbol{R}^3$ which is defined as follows:

(1) $$R_\theta(t)x = (x_1(t), x_2(t), x_3(t)), \qquad x \in X_0,$$

where $x_1(t) = r(\cos\theta\cos(\theta-\alpha) + \sin\theta\sin(\theta-\alpha)\cos t)$, $x_2(t) = r(\sin\theta\cos(\theta-\alpha) - \cos\theta\sin(\theta-\alpha)\cos t)$, $x_3(t) = r\sin t\sin(\theta-\alpha)$ and $x = r(\cos\alpha, \sin\alpha, 0)$, $r = |x|$, $0 \leqq \alpha < 2\pi$. The operation $x \mapsto R_\theta(t)x$ geometrically means the rotation of x around the line l_θ: $-x_1\tan\theta + x_2 = x_3 = 0$ in t-degree. From the definition of $R_\theta(t)$ we immediately see that

(2) $R_\theta(t)$ is linear and isometric for any $t \in \boldsymbol{R}$ and $\theta \in [0, \pi)$, and

(3) $R_\theta(t)$ is a C^∞-function of t for any $x \in X_0$ and $\theta \in [0, \pi)$.

For the moment we fix a number θ with $0 < \theta < \pi$. For each $t \in \boldsymbol{R}$ we define the operator $S(t)$ from X_0 into $\boldsymbol{R}^3$ by

(4) $$S(t) = S_0(t - 2n\pi)[S_0(2\pi)]^n \qquad \text{for } t \in [2n\pi, 2(n+1)\pi),\ n \in \boldsymbol{Z},$$

where

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