92. On Some Algebraic Differential Equations with Admissible Algebroid Solutions

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1. Introduction. About fifty years ago, K. Yosida ([9]) proved the following theorem.

Theorem A. *When the differential equation with rational coefficients*

$$(w')^m=\sum_{j=0}^{p} a_j w^j \Big/ \sum_{k=0}^{q} b_k w^k \qquad (a_p\cdot b_q\neq 0),$$

where m is a positive integer and $\sum a_j w^j$, $\sum b_k w^k$ are irreducible, admits at least one transcendental ν-valued algebroid solution in $|z|<\infty$, then it holds that

(1) $$\max(p, q+2m)\leqq 2m\nu.$$

This theorem was extended by several authors ([1], [2], [3], [4] etc.). In this paper, we shall consider the differential equation

(2) $$\Omega(w, w', \cdots, w^{(n)})=P(w)/Q(w),$$

where $\Omega(w, w', \cdots, w^{(n)})=\sum_{\lambda\in I} c_\lambda w^{i_0}(w')^{i_1}\cdots(w^{(n)})^{i_n}$ $(n\geqq 1)$ is a differential polynomial with meromorphic coefficients, I being a finite set of multi-indices $\lambda=(i_0, i_1, \cdots, i_n)$, ($i_l$: non-negative integers), for which $c_\lambda\neq 0$, and where $P(w)$, $Q(w)$ are polynomials in w with meromorphic coefficients and mutually prime over the field of meromorphic functions:

$$P(w)=\sum_{j=0}^{p} a_j w^j \quad (a_p\neq 0), \qquad Q(w)=\sum_{k=0}^{q} b_k w^k \quad (b_q\neq 0).$$

The term "meromorphic" (resp. "algebroid") will mean meromorphic (resp. algebroid) in the complex plane. Put

$$\Delta=\max_{\lambda\in I}\sum_{j=0}^{n}(j+1)i_j, \quad \Delta_o=\max_{\lambda\in I}\sum_{j=1}^{n} j i_j, \quad d=\max_{\lambda\in I}\sum_{j=0}^{n} i_j$$

and

$$\sigma=\max_{\lambda\in I}\sum_{j=1}^{n}(2j-1)i_j.$$

An algebroid solution $w=w(z)$ of (2) is said to be admissible when $T(r, f)=S(r, w)$ for all coefficients $f=c_\lambda$, a_j and b_k in (2), where $S(r, w)$ is any quantity satisfying $S(r, w)=o(T(r, w))$ as $r\to\infty$, possibly outside a set of r of finite linear measure.

Recently, Gackstatter and Laine ([1], [2]), Y. He and X. Xiao ([3]) extended Theorem A as follows:

"*If the differential equation* (2) *admits an admissible algebroid solution $w=w(z)$ with ν branches, then*

(i) $q\leqq 4\Delta_o(\nu-1)$, $p\leqq \Delta+4\Delta_o(\nu-1)$ ([1], [2]),

(ii) $q\leqq 2\sigma(\nu-1)$, $p\leqq q+d+\Delta_o\nu(1-\theta(w, \infty))$ ([3])

where $\theta(w, \infty)=1-\limsup_{r\to\infty}\overline{N}(r, w)/T(r, w)$."

In this paper, we shall improve these results and give some examples.

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