85. *A Nonsymmetric Partial Difference Functional Equation Analogous to the Wave Equation*

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§1. **Introduction.** The purpose of this note is to announce the general solution of the nonsymmetric partial difference functional equation

$$\text{(N)}\qquad \frac{f(x+t, y)+f(x-t, y)-2f(x, y)}{t^2}=\frac{f(x, y+s)+f(x, y-s)-2f(x, y)}{s^2}$$

analogous to the well-known wave equation

$$\left(\frac{\partial^2}{\partial x^2}-\frac{\partial^2}{\partial y^2}\right)f(x, y)=0$$

with the aid of generalized polynomials when no regularity assumptions are imposed on f.

Let R be the set of all real numbers, and let f be a function on the plane $R\times R$ taking values in R. Define the divided symmetric partial difference operators $\underset{x,t}{\triangle}$ and $\underset{y,t}{\triangle}$ by

$$(\underset{x,t}{\triangle} f)(x, y)=[f(x+t/2, y)-f(x-t/2, y)]/t$$
$$(\underset{y,t}{\triangle} f)(x, y)=[f(x, y+t/2)-f(x, y-t/2)]/t$$

for all $x, y\in R$ and for all $t\in R\backslash\{0\}$.

The symmetric partial difference functional equation

$$((\underset{x,t}{\triangle}^2-\underset{y,t}{\triangle}^2)f)(x, y)=0$$

analogous to the wave equation or, in expanded form,

$$f(x+t, y)+f(x-t, y)=f(x, y+t)+f(x, y-t)$$

for all $x, y, t\in R$ has been studied by J. Aczél, H. Haruki, M. A. McKiernan and G. N. Sakovič [1], J. A. Baker [2], D. P. Flemming [3], D. Girod [4], H. Haruki [5], M. Kucharzewski [7], M. A. McKiernan [10], and others.

In this note we will consider the nonsymmetric partial difference functional equation

$$((\underset{x,t}{\triangle}^2-\underset{y,s}{\triangle}^2)f)(x, y)=0$$

which is equivalent to the above expanded form (N) for all $x, y\in R$ and for all $s, t\in R\backslash\{0\}$ and $s\neq t$. Equation (N) is stated in [3] without finding a solution.

§2. **The general solution of (N).** The result is as follows.

Theorem 1. *A function $f: R\times R\rightarrow R$ satisfies functional equation* (N) *for all $x, y\in R$, $s, t\in R\backslash\{0\}$, and $s\neq t$ if and only if there exist*

(i) *additive functions $A, B: R\rightarrow R$,*

(ii) *a function $C: R\times R\rightarrow R$ which is additive in both variables, and*