67. *Quadratic Spline Interpolation on a Jordan Curve*

By Aruna CHAKRABARTI
Jadavpur University, India

(Communicated by Shokichi IYANAGA, M. J. A., Sept. 12, 1985)

1. Summary. The existence, uniqueness and convergence properties of quadratic splines interpolating to a given function $f(z(t))$ at an intermediate point of each subarc have been studied.

2. Existence and uniqueness. Let I be the interval $[0,1]=\{t: 0\leqslant t \leqslant 1\}$, $\Delta=\{t_0, t_1, \cdots, t_n\}$, $0=t_0<t_1<\cdots<t_n=1$ a subdivision of I and $I_j=[t_{j-1}, t_j]$, the j-th subinterval of I. Let $K=\{z(t): t\in I\}$, $z(0)=z(1)$, be a closed Jordan curve and $K_j=\{z(t): t\in I_j\}$ the j-th subarc of K corresponding to Δ. Let furthermore λ be a number $\in(0,1)$. Put $f(z(t))=F(t)$, $h_j=t_j-t_{j-1}$ and $\alpha_j=t_{j-1}+\lambda h_j$ for $j=1,2,\cdots,n$ so that $z(\alpha_j)\in K_j$. Considering $q_\Delta(t)\in C^1(I)$ with the interpolatory condition

$$q_\Delta(\alpha_j)=F(\alpha_j) \qquad j=1,2,\cdots,n, \tag{2.1}$$

we shall prove the following:

Theorem 2.1. *If $f(z(t))$, $t\in I$, be a given functiton on K, then there exists a unique periodic quadratic spline $q_\Delta(t)\in C^1(I)$ satisfying the interpolatory condition* (2.1).

Proof of Theorem 2.1. Let $P(t)=(t-t_j)(t-t_{j-1})(t-\alpha_j)$. We suppose that in I_j,

$$q_\Delta(t)=AP_j(t)-BP_{j-1}(t)-CP_j(t,\alpha) \tag{2.2}$$

where $P_i(t)$ $(i=j, j-1)$ is $P(t)$ without $(t-t_i)$ and $P_j(t,\alpha)$ is $P(t)$ without $(t-\alpha_j)$ (cf. [2]).

Writing $q'_\Delta(t_j)=M_j$, $j=1,2,\cdots,n$, and using (2.1) we have from (2.2)

$$M_jh_j^{-1}=(2-\lambda)A-(1-\lambda)B-F_j(\alpha,h;\lambda) \tag{2.3}$$

$$M_{j-1}h_j^{-1}=-\lambda A+(1+\lambda)B+F_j(\alpha,h;\lambda) \tag{2.4}$$

where

$$F_j(\alpha,h;\lambda)=\lambda^{-1}(1-\lambda)^{-1}h_j^{-2}F(\alpha_j). \tag{2.5}$$

Using (2.3)–(2.4), we get another expression for $q_\Delta(t)$:

$$\begin{aligned}2q_\Delta(t)=&M_{j-1}h_j^{-1}((1-\lambda)P_j(t)-(2-\lambda)P_{j-1}(t))\\&+M_jh_j^{-1}((1+\lambda)P_j(t)-\lambda P_{j-1}(t))\\&+2F_j(\alpha,h;\lambda)(\lambda P_j(t)+(1-\lambda)P_{j-1}(t)-P_j(t,\alpha)).\end{aligned} \tag{2.6}$$

Since $q_\Delta(t_j-)=q_\Delta(t_j+)$, $j=1,2,\cdots,n$; we get

$$\begin{aligned}(1-\lambda)^2a_jM_{j-1}+((1-\lambda^2)a_j+(2\lambda-\lambda^2)b_j)M_j+\lambda^2b_jM_{j+1}\\=2(h_j+h_{j+1})^{-1}(F(\alpha_{j+1})-F(\alpha_j))\end{aligned} \tag{2.7}$$

where

$$a_j=h_j/(h_j+h_{j+1}) \quad\text{and}\quad b_j=1-a_j. \tag{2.8}$$

The existence and uniqueness of the spline $q_\Delta(t)$ rests upon the existence of a unique solution of the equations (2.7) in M_j's. This follows if