6. *Gauss Sums of Prehomogeneous Vector Spaces*[*)]

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In the present article, we study a generalization of the classical Gauss sum which is associated with a prehomogeneous vector space, by using the micro-local analysis. Details which are omitted here will be published elsewhere.

1. Let V be a finite dimensional vector space over $\boldsymbol{C}$ and G a connected algebraic subgroup of $GL(V)$ which acts prehomogeneously on V, that is, there exists a proper algebraic subset S of V such that G acts homogeneously on $V-S$. We call such a pair (G, V) a prehomogeneous vector space. (See [12].) Hereafter we assume the following two conditions:

(1.1) G acts irreducibly on V.

(1.2) S is an (irreducible) hypersurface of V, that is, there exists an irreducible polynomial $f(v)$ such that $S=\{v \in V \mid f(v)=0\}$.

Such a prehomogeneous vector space is said to be *irreducible* and *regular*. Let $V^\vee$ be the dual space of V. Then $(G, V^\vee)$ is also an irreducible, regular prehomogeneous vector space. We define $S^\vee$ and $f^\vee$ in the same way as S and f. Let $\langle\, ,\rangle$ be the natural pairing of $V^\vee$ and V. Let $V^\vee \xleftarrow{\mathrm{pr}^\vee} V^\vee \times V \xrightarrow{\mathrm{pr}} V$ be the projections and $j: V-S \to V$, $j^\vee: V^\vee - S^\vee \to V^\vee$ the inclusion mappings. Let $n=\dim V$ and $d=\deg f$. It is known that there exists a polynomial $b(s)$ such that

$$f^\vee(\mathrm{grad}) f^{s+1} = b(s) f^s.$$

(See [12].) It is also known that $b(s)$ is of the form

$$b(s)=b_0 \prod_{j=1}^{d} (s+\alpha_j) \qquad (\alpha_j \in \boldsymbol{Q}, \alpha_j > 0),$$

([6]). Let

$$b^{\exp}(t) = \prod_{j=1}^{d} (t-\exp(2\pi\sqrt{-1}\,\alpha_j)).$$

Then we can show that

$$b^{\exp} = \prod_{l \geq 1} \Phi_l^{m(l)} \tag{1.3}$$

with some non-negative integers $m(l)$. Here Φ_l denotes the l-th cyclotomic polynomial.

2. By a classification [12] of irreducible, regular prehomogeneous vector spaces, we see that (G, V) has a natural $\boldsymbol{Z}$-structure. If p is a sufficiently large prime number, we can get an irreducible, regular prehomogeneous vector space defined over $\boldsymbol{F}_p$, by the reduction modulo p, which

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