48. Continuum of Ideals in $R(\Phi_2)\otimes_{\max} R'(\Phi_2)$

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Let Φ_2 be the free group on two generators a and b. Let $\mathcal{H}=\mathcal{L}^2(\Phi_2)$ be the Hilbert space of all complex valued functions $f(g)$ on Φ_2 such that

$$\sum_{g\in\Phi_2}|f(g)|^2<\infty.$$

For each $g_1\in\Phi_2$ we define the unitary operator $U(g_1)$ on $\mathcal{H}$ given by

$$(U(g_1)f)(g)=f(g_1^{-1}g),\qquad \text{for all } f\in\mathcal{H}.$$

The von Neumann algebra generated by $\{U(g),\ g\in\Phi_2\}$ is denoted by $R(\Phi_2)$. It is known that $R(\Phi_2)$ is a II_1-factor.

The purpose of this paper is to show the existence of continuum of ideals in $R(\Phi_2)\otimes_{\max} R'(\Phi_2)$.

We will use the following universal property of the projective C^*-tensor product.

Lemma 1. *Given C^*-algebras A_1, A_2 and B, if $\pi_1: A_1\to B$ and $\pi_2: A_2\to B$ are homomorphisms with commuting ranges, then there exists a unique homomorphism π of the projective C^*-tensor product $A_1\otimes_{\max} A_2$ into B such that*

$$\pi(x_1\otimes x_2)=\pi_1(x_1)\pi_2(x_2)\qquad x_1\in A_1,\ x_2\in A_2,$$

and the image $\pi(A_1\otimes_{\max} A_2)$ is the C^*-subalgebra of B generated by $\pi_1(A_1)$ and $\pi_2(A_2)$ (cf. [4, p. 207]).

We denote by $\mathrm{Int}(R(\Phi_2))$ and $\mathrm{Aut}(R(\Phi_2))$ the set of all inner automorphisms and that of all automorphisms of $R(\Phi_2)$ respectively, with the topology of strong pointwise convergence in $R(\Phi_2)$.

Lemma 2. $\mathrm{Int}(R(\Phi_2))$ *is closed in* $\mathrm{Aut}(R(\Phi_2))$.

For the proof see [3, Corollory 3.8].

In the following we will use the Connes's characterization of approximately inner automorphisms.

Lemma 3. *Let N be a factor of type* II_1 *with separable predual acting in $\mathcal{K}=L^2(N,\tau)$. Then the following conditions are equivalent for $\theta\in\mathrm{Aut}(N)$,*

(a) $\theta\in\overline{\mathrm{Int}(N)}$;

(b) *There exists an automorphism of the C^*-algebra generated by N and N' in $\mathcal{K}$ which is θ on N and identity on N'* ([2, p. 89]).

In Lemma 1, if we put $A_1=R(\Phi_2)$, $A_2=R'(\Phi_2)$ and π_1, π_2 as identical map, there exists a homomorphism η such that

$$R\underset{\max}{\otimes}R'\xrightarrow[\text{onto}]{\eta}C^*(R,R'),$$

$$R\underset{\max}{\otimes}R'/I\cong C^*(R,R')$$

in which I is $\mathrm{Ker}(\eta)$.