# 43. A Formula of Eigenfunction Expansions II.

## Exterior Dirichlet Problem in a Lattice

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We apply the method used in my previous note to exterior Dirichlet problems in a lattice. It is shown there is no point spectrum.

**1.** Let $\Gamma$ be a free abelian group with $g$ generators $\sigma_1, \cdots, \sigma_g$ and $A_0$ be a self-adjoint bounded linear operator on $l^2(\Gamma)$ described by a symmetric stochastic walk on $\Gamma$:

$$A_0 u(\gamma)=\sum_{i=1}^{g} p_i(u(\gamma\sigma_i)+u(\gamma\sigma_i^{-1})). \tag{1.1}$$

Let $A$ be the restriction of $A_0$ on $l^2(\Gamma-\Omega)$ corresponding to the exterior Dirichlet problem outside a finite subset $\Omega$. Physically this corresponds to a random walk with traps $\Omega$ (see [5]). The Green function for $A_0$ is described by the Fourier integral formula

$$G_0(\gamma,\gamma'\,|\,z)=\frac{1}{(2\pi i)^g}\int_{S^1\times\cdots\times S^1}\frac{\omega_1^{-m_1+m_1'}\cdots\omega_g^{-m_g+m_g'}}{z-\sum_1^g p_j(\omega_j+\omega_j^{-1})}\cdot\frac{d\omega_1}{\omega_1}\wedge\cdots\wedge\frac{d\omega_g}{\omega_g} \tag{1.2}$$

for $\gamma=\sigma_1^{m_1}\cdots\sigma_g^{m_g}$ and $\gamma'=\sigma_1^{m_1'}\cdots\sigma_g^{m_g'}$ where $z\in \boldsymbol{C}-[-1,1]$. The integral depends only on $|m_1-m_1'|, \cdots, |m_g-m_g'|$.

Let $S^{g-1}$ be the unit sphere of dimension $g-1$ and $S^{g-1}(\varepsilon_1,\cdots,\varepsilon_g)$ be the quadrant of $S^{g-1}$ consisting of points $(\xi_1,\cdots,\xi_g)\in S^{g-1}$ such that $\varepsilon_1\xi_1>0, \cdots, \varepsilon_g\xi_g>0$ for $\varepsilon_j=\pm 1$. We denote by $V_z$ the analytic hypersurface (so called complex Fermi hypersurface) in $(\boldsymbol{C}^*)^g$ defined by

$$F(z,\omega,\omega^{-1})\equiv z-\sum_{j=1}^{g} p_j(\omega_j+\omega_j^{-1})=0. \tag{1.3}$$

For a given direction at infinity $\xi=(\xi_1,\cdots,\xi_g)\in S^{g-1}(\varepsilon_1,\cdots,\varepsilon_g)$ consider the following equation with respect to the variables $\omega_j=\exp(\sqrt{-1}\theta_j)$ which is the inverse of the Gauss map $\kappa$ from $V_z$ to $S^{g-1}$:

$$\frac{1}{i}\frac{\partial F}{\partial\theta_j}\left(\equiv\omega_j\frac{\partial F}{\partial\omega_j}\right)=\xi_j\rho, \qquad 1\leqq j\leqq g \tag{1.4}$$

for an unknown $\rho$. This has the following solution $\hat{\omega}=(\hat{\omega}_1,\cdots,\hat{\omega}_g)\in V_z$:

$$\hat{\omega}_j=\frac{-\varepsilon_j\xi_j\rho+\sqrt{(\rho\xi_j)^2+4p_j^2}}{2p_j} \tag{1.5}$$

where $\rho$ denotes the unique solution of the equation

$$\sum_{j=1}^{g}\sqrt{\zeta_j^2+4p_j^2}=z \qquad \text{for } \zeta_j=\xi_j\rho \tag{1.6}$$

such that $\rho>0$ for $z>1$.

By saddle point method and Lagrangean analysis for the Hamiltonian $\mathrm{I}_m \sum_1^g m_j'\log\omega_j$ in the Kähler manifold $V_z$ ([1]), we can prove