35. A New Formulation of Local Boundary Value Problem in the Framework of Hyperfunctions. II

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This is a continuation of our previous paper [4]. In it we formulated non-characteristic boundary value problems for systems of linear partial differential equations and proved a Holmgren's type uniqueness theorem.

Here we first clarify the meaning of boundary values of hyperfunction solutions in the non-characteristic case by using F-mild hyperfunctions. Next we study boundary value problems for partial differential equations with regular singularities from our viewpoint apart from that of Kashiwara-Oshima [2]. Finally we microlocalize these boundary value problems in order to study micro-analyticity of solutions near the boundary.

We use the same notation as in [4]:

$$M=\boldsymbol{R}^n \ni x=(x_1, x'), \quad X=\boldsymbol{C}^n \ni z=(z_1, z'), \quad z'=(z_2, \cdots, z_n),$$
$$N=\{x \in M\,;\, x_1=0\}, \quad Y=\{z \in X\,;\, z_1=0\}, \quad \tilde{M}=\boldsymbol{R}\times\boldsymbol{C}^{n-1},$$
$$M_+=\{x \in M\,;\, x_1 \geqq 0\}, \qquad \operatorname{int} M_+=\{x \in M\,;\, x_1>0\}.$$

We set $\mathcal{B}_{N|M_+}=(\iota_*\iota^{-1}\mathcal{B}_M)|_N$, where $\mathcal{B}_M$ is the sheaf of hyperfunctions on M and $\iota: \operatorname{int} M_+ \to M$ is the natural embedding.

§ 1. Non-characteristic boundary value problems. First let us recall the definition of F-mild hyperfunctions.

Definition 1 (Ôaku [5]). Let f be a germ of $\mathcal{B}_{N|M_+}$ at $\mathring{x} \in N$. Then f is called F-mild at $\mathring{x}$ if and only if f has a boundary value expression

$$f(x)=\sum_{j=1}^{J} F_j(x_1, x'+\sqrt{-1}\,\Gamma_j 0)$$

as a hyperfunction on $\{x \in \operatorname{int} M_+\,;\, |x-\mathring{x}|<\varepsilon\}$, where J is a positive integer, ε is a positive number, Γ_j is an open convex cone, F_j is a holomorphic function defined on a neighborhood (in $\boldsymbol{C}^n$) of

$$\{z=(z_1, z') \in \boldsymbol{C}^n\,;\, |z-\mathring{x}|<\varepsilon,\ \operatorname{Re} z_1 \geqq 0,\ \operatorname{Im} z_1=0,\ \operatorname{Im} z' \in \Gamma_j\}.$$

For an open set U of N, $\mathcal{B}^F_{N|M_+}(U)$ denotes the set of sections of $\mathcal{B}_{N|M_+}$ over U which are F-mild at each point of U. Then $\mathcal{B}^F_{N|M_+}$ is a subsheaf of $\mathcal{B}_{N|M_+}$ and called the sheaf of F-mild hyperfunctions. We denote by $\mathcal{D}_X$ the sheaf of rings of linear partial differential operators with holomorphic coefficients on X.

Theorem 1. *Let $\mathcal{M}$ be a coherent $\mathcal{D}_X$-module for which Y is non-characteristic. Then we have*

$$\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{B}_{N|M_+}/\mathcal{B}^F_{N|M_+})=0,$$

and in particular

$$\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{B}^F_{N|M_+})=\mathcal{H}om_{\mathcal{D}_X}(\mathcal{M}, \mathcal{B}_{N|M_+}).$$